TO

LIEUT. COL. SIR JOHN RAMSAY, K.C.I.E., C.S.I.,

LATE AGENT TO THE GOVERNOR-GENERAL AND

CHIEF COMMISSIONER, BALUCHISTAN,

AND TO

MY RESPECTED FATHER

HABIBUR RAHMAN KHAN, K.B., M.I.E., ETC.,

DISCOVERER OF WATER WIRELESS TELEGRAPHY.

THE

SYMBOLIC METHOD

AND

ITS APPLICATION TO ALTERNATING CURRENT CIRCUITS

BY

ABDUL GHAFOOR KHAN, B A (Alig)

B Sc (Honours) M Sc (Tech) A M I E E (London) etc

ELECTRICAL ENGINEER P W D IMPERIAL DELHI

LATE GOVERNMENT OF INDIA (BALUCHISTAN) TECHNICAL SCHOLAR MANCHESTER UNIVERSITY COLLEGE OF TECHNOLOGY RESEARCH SCHOLAR SOMETIME RESEARCH ENGINEER TO THE HIGH TENSION RESEARCH PANEL OF THE INSTITUTION OF ELECTRICAL ENGINEERS (LONDON) FORMERLY WORKED IN THE LANCASHIRE DYNAMO AND MOTOR WORKS MANCHESTER IN THE PLANT TRANSFORMER SWITCHGEAR AND CONTROLGEAR WORKS AND IN THE COMMERCIAL DEPARTMENT OF THE METROPOLITAN VICKERS ELECTRICAL COMPANY MANCHESTER SOMETIME WITH MESSRS GWYNNES LTD HYDRAULIC ENGINEERS HAMMERSMITH LONDON AND THE SOUTH WALES POWER DISTRIBUTION CO CARDIFF

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The following books, by the same author, are in preparation and will shortly be published —

1 Financial and Technical Organisation and Management of Electric Power Generation, Distribution and Supply

2 The Predetermination of the Economics of Electrical Power Supply for Industrial and Domestic purposes

3 The Predetermination of the Operating Characteristics of long distance Hydro Electric Transmission Lines

4 Long distance Hydro Electric Transmission Lines in relation to Wireless Telegraphy

5 The Automatic, Contactor type, Controlgear for Industrial Motors

6 The standardized types of Electric Motor and Switchgear, their Commercial and Engineering Characteristics and Applications to Industry

PREFACE

In the following pages, I have made an attempt to describe the Symbolic Method and its application to alternating current circuits. The attempt has been made, primarily, for the benefit of the electrical student and the embryo electrical Engineer in India who may not have suitable facilities, near at home, for the study of the subject.

The Symbolic Method is now being taught in most of the first rate University grade technical institutions of Great Britain. The importance of the method can be judged from the fact that in London, Manchester, and other great Industrial centres, a series of evening lectures on the subject are given annually for the benefit of those who can not attend the day classes. I strongly hope that Colleges in India, teaching Electrical Engineering and Wireless Telegraphy shall also introduce the study of the Symbolic Method. The Method enables the student to enter *mentally* into the phenomena and behaviour of alternating current circuits, and

also assists him in forming a clear mental picture of how alternating currents and voltages behave when flowing in circuits containing Resistance, Inductance, and Capacity

The application of the Symbolic Method can be carried out in several ways, but the system I have adopted in the book has many points in its favour The chief advantage being that current and voltage vectors remain in their analytical form, and the conversion of the one to the other is carried out by the operation of the impedance vector which alone is to be expressed symbolically

With a view to increase the practical usefulness of the work, I have omitted reference to the historical aspect of the method and also a great deal of information relating to the theory of alternating current The latter can be found in all standard alternating current books The first three chapters, therefore, are intended merely to draw the attention of the student to the fundamentals of alternating current I have worked out in the last chapter, in full detail, several numerical examples illustrating the application of the Symbolic Method to alternating current circuits, and an industrious student should have no difficulty in learning the subject at home

I am deeply indebted to my professors from whom I had, initially, learnt the Symbolic Method, and to authors on alternating current and to the Exponents of the Symbolic Method to whose teachings also I owe a very great deal

A G KHAN

10 Parliament Street,
Raisina Delhi
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THE SYMBOLIC METHOD

AND

ITS APPLICATION TO ALTERNATING CURRENT CIRCUITS

CHAPTER I

THE ELECTRIC CURRENT

1 **Two Classes of Electric Currents**—The Electrical Engineer has to deal, in the practical applications of Electricity, with two classes of Electric Currents, namely

(*a*) The Direct (Continuous) Current

(*b*) The Alternating Current

2 **Flow of Current**—We know that whenever a difference of Potential exists in a closed electric circuit, a current of electricity *always* flows from a point of higher to a point of lower potential, and the direction of the flow of current remains *unaltered* as long as the direction of the flow of potential remains unchanged

3 **The Direct Current**—The voltage given out by a voltaic cell, a battery, or a dynamo is such that its respective positive and negative polarities do not undergo periodic reversals If such a voltage is applied across a closed

electric circuit, the current will flow in one direction only, and such a current is known as a *direct current* Its graph, therefore, would consist of a straight line such as that shown in Fig 1

TIME ——

FIG 1

Graph of Direct Current at Constant Volts and Constant Resistance

4 **The Alternating Current**—The voltage generated by an alternating current generator and available at its slip rings not only undergoes a periodic reversal in polarity, but that it changes also in value from instant to instant This class of voltage is known as an *alternating* voltage If it is applied across a simple closed circuit, it is quite evident that the current driven by the voltage will not only change its direction of flow at fixed and constantly recurring intervals, but that it will vary in strength from instant to instant It can be understood best by a study of the graphs of Fig 2 We note from the graphs, that the alternating current rises from zero to a maximum and then falls to zero It again changes its value in the same way but in the reverse direction Such a

current is known as an *Alternating Current* It will be defined as one which not only changes its direction of flow at fixed and constantly recurring intervals, but that it varies also in strength from instant to instant

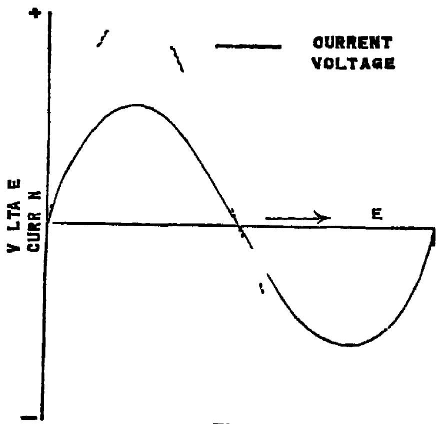


FIG 2

Graphs of Current and Voltage Sine Waves

3 Characteristics of Alternating Currents

The variation of the alternating current from an instant when the current is zero and is to rise in the positive direction, to the instant when it is again zero and is to rise again in the positive direction, is called a *Cycle* The number of such Cycles per second is termed *Frequency* It is therefore clear that an alternating current follows a cycle of variation and these cycles have a certain frequency per second The characteristic of an alternating current is that it will have the same frequency as that of the voltage which is driving the current The alternating current may be or may not be 'in phase' with its voltage In Figure 2 we have taken a case where the current keeps time with the voltage,

i e it is zero when the voltage is zero and it is maximum when the voltage is maximum Its phase position, relative to the impressed voltage, will depend, as we shall observe later, upon the *properties* of the Circuit We would also observe later that if the circuit contained only pure *Ohmic resistance* the alternating current would be in phase with its voltage, but if it contained *Inductance* or *Capacity* the current might not remain in phase with the voltage

6 **Measurement of Alternating Currents** — A current of Electricity, flowing through a circuit, is generally known by its effects In a circuit carrying direct current, the effect produced, which will be proportional to the square of the current, will be the same from instant to instant since the current remains constant in value When the current is an alternating one, the effect produced by it will also be proportionate to the square of the current, with this difference that the effect will vary from instant to instant because the strength of the current varies from instant to instant

It is therefore evident that the *direct* and the *alternating* currents will be the same, as regards their effects, if the mean of the squares of the two currents at any instant, during a certain period is the same In other words

if the square root of the mean of the squares (*i e* the root mean square value briefly termed R M S value) of the direct current at any instant during a certain period is the same as the square root of the mean of the squares of the alternating current at any instant during a complete cycle of variations

The strength of the direct current is the same at every instant and therefore the root of the mean square of the values of direct current from instant to instant over a period is the same as the value of the current at any instant It is the alternating current whose root mean square value (R M S value) is not the same as the instantaneous value It is clear, therefore, that in the practical applications of alternating current, the numerical value commonly referred to is the R M S value

7 **Numerical Value of an Alternating Voltage and Current**

In practice the wave form of the voltage of an alternating current generator (commonly known as Alternator) is assumed to approximate to a sine wave

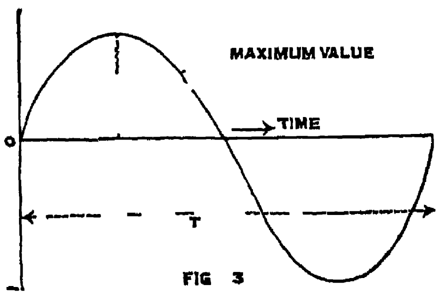


FIG 3

Graph of Sine Wave Current or Voltage

form as shown in Fig 5

It can be expressed, analytically, by an equation such as

$$e = E \sin pt \qquad (1)$$

where e is the instantaneous value at any time t and E is the maximum value of the voltage sine wave

If we measure the area of the graph of the square of the function over a period and divide it by the base line T, we shall get the mean of the squares of the function over a period and the square root of it will give us the R M S value which would be found to be equal to

$\frac{1}{\sqrt{2}}$ i e $.707 \times E$ or $.707 \times$ the maximum value

We get the above value, mathematically, as follows —

Area of the graph of the square of the function over a period T

$$= \int_0^T E^2 \sin^2 pt\, dt = \int_0^T E^2 \times \tfrac{1}{2}(1 - \cos 2pt)\, dt$$

$$= \tfrac{1}{2}\int_0^T E^2\, dt - \tfrac{1}{2}\int_0^T E^2 \cos 2pt\, dt$$

$$= \tfrac{1}{2} E^2 (t)_0^T - \tfrac{1}{4} E^2 (\sin 2pt)_0^T$$

$$= \tfrac{1}{2} E^2 T - \tfrac{1}{4} E^2 \sin 2p\, T \qquad (2)$$

Since T is the time taken by the wave to pass through a complete cycle, i e to pass through 2π radians,

$$\sin 2\,pT = 0$$

therefore equation (2) becomes

$$\tfrac{1}{2} E\ T$$

Therefore the area $= \frac{1}{2} E\ T$

and dividing this area by the base line T, we get $\frac{E}{2}$ as the mean of the squares of the instantaneous values

$$\text{R M S value} = \sqrt{\frac{E}{2}} = \frac{E}{\sqrt{2}} = 707\ E$$

$= 707 \times$ maximum value of the wave

It should be noted that in the practical application of alternating current, we generally refer to the R M S value and this is always taken to be $707 \times$ maximum value

The above relation could be established in a simpler way, not involving the use of Integral Calculus, which is as follows —

The alternating current which is expressed analytically by an equation $i = I \sin pt$ can be represented vectorially by a rotating vector OP, (Fig 4), of length I (the maximum value of the current), revolving at an angular velocity of p radians per second The instantaneous value i, which is equal to $I \sin pt$ will be

seen from Fig 4, to be equal to PM, i.e. to the vertical projection of the rotating vector

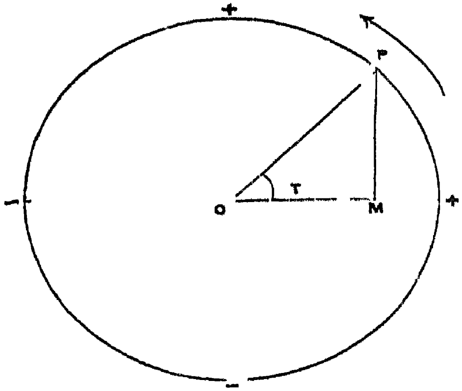


FIG 4

Vector diagram

The extremity of the vertical projection of the rotating vector moves with a simple harmonic motion between the values zero and the maximum value I And the extremity of the horizontal projection of the rotating vector also moves with a simple harmonic motion between the values zero and I And the mean of the squares of the vertical projections of the rotating vector OP = the mean of the squares of its horizontal projections

The vertical projection of the rotating vector I is I $\sin pt$ and the horizontal projection of I is I $\cos pt$

Therefore mean value of $I^2 \sin^2 pt$

= mean value of $I^2 \cos^2 pt$

$= \frac{1}{2}$ (mean value of $I^2 \sin^2 pt$ + mean value of $I^2 \cos^2 pt$)

$= \frac{1}{2} I^2 \times$ mean value of $(\sin^2 pt + \cos^2 pt) = \frac{1}{2} I^2$

Since $\sin^2 pt + \cos^2 pt = 1$

∴ mean value of the squares of the instantaneous values of the current $= \frac{1}{2} I^2$ where I is the maximum value

∴ square root of the mean value of the squares of the instantaneous values of the current,

i.e. the R. M. S. value $= \dfrac{I}{\sqrt{2}} = .707 \times I$

It should be clearly understood that when we refer to the strength of the alternating voltage or current, we really mean the effective value, *i.e.* the R. M. S. value of the voltage or current.

From the foregoing we derive the following meanings of the *strength* of an alternating current :—

(*a*) The strength of an alternating current is really the root mean square (*i.e.* R. M. S.) value of the alternating current.

(*b*) In order that an alternating current may develop the same power as a direct current of say C amperes, the strength of the alternating current, *i.e.* the Root mean square value of the alternating current,

must also be C amperes, in which case the maximum value of the alternating current will be $\frac{C}{707}$ amperes.

(c) In order that an alternating current may develop the same power as a direct current, the mean value of the squares of the instantaneous values of the alternating current must equal the square of the strength of the direct current, i.e. ½ of the square of the maximum value of the alternating current must equal the square of the direct current.

CHAPTER II

PHYSICAL CONDITION OF THE PROPERTIES OF ELECTRIC CIRCUITS

8 **The Three Properties** —Every Electric Circuit possesses some or all of the following three properties —

1 Ohmic Resistance
2 Inductance
3 Capacity

9 **Ohmic Resistance** —All electric conductors offer a certain amount of resistance to the flow of electricity It is due to some sort of molecular friction in the conductor itself and its magnitude is determined by the cross section, length of the conductor and the material of which it is made and also the temperature of the material In direct current circuits, the ohmic resistance *alone** is the determining factor in the strength of the current, but in alternating current circuits, the ohmic resistance ceases to be the *only* determining factor on account of part played by inductance and capacity

* It must be remembered that inductance *does* manifests itself in circuits carrying direct current, but only at moments of make and break

10 **Inductance**.—The second property which an electric circuit may possess, but which manifests itself only when the circuit is carrying alternating current, is known as "Inductance." It can be defined as that property of the circuit which opposes any *change* in the flow of electric current. It is due to the magnetic field which becomes associated with the conductor when it is carrying a current. Its value, therefore, is determined by the shape of the conductor, and the nature of its immediate surroundings, *i.e.* presence of iron, etc. As we shall observe later, inductance of the circuit becomes one of the determining factors of the value of current flowing in the circuit when the applied voltage is *alternating*.

In an electric circuit carrying a direct current no change of flux, associated with the conductor, takes place so long as the current strength remains constant and therefore no back e.m.f. of induction is generated in the circuit. It is only at the moments of switching on and off of the direct current that a change takes place in the flux and the conductor becomes the seat of the induced e.m.f. When the direct current is switched on, the induced e.m.f. has such a direction that it opposes the applied

voltage and therefore retards the growth of the current strength. At the moment of switching off the current, the vanishing field induces an e. m. f. in a direction that it helps the applied voltage and thus tends to maintain the current in the circuit. So this phenomenon becomes apparent in the case of direct current during the moments of switching on and off only.)

When the electric circuit is carrying an alternating current, the magnetic flux linked round the circuit will also be alternating and this varying flux will induce an alternating e. m. f. which will tend to oppose any *change* in the flow of current. It is the consequence of the electro magnetic properties of an electric circuit that we observe the phenomenon that a circuit carrying alternating current shows a tendency to *oppose* any change in the flow of the current. This phenomenon, which is an inherent property of an electric circuit and which expresses its presence seriously, only when the circuit is carrying an alternating current, is known under the name of "Inductance". In all practical applications of alternating currents the "Inductance" property of the circuit plays such an important part that it is essential that the student should form a clear mental picture of

the nature and character of this property

11 **Mechanical Analogy to illustrate Inductance** —We may borrow a mechanical analogy to further illustrate this property We know that all bodies show a tendency to oppose a change either from a state of rest to that of motion or *vice versa* The tendency of stationary body to oppose being put into motion is attributable to its "Inertia", and the tendency of a moving body to oppose being stopped is due to its "Momentum" We experience very similar phenomenon in the case of an electric circuit When a circuit is carrying a current, it shows a tendency to *oppose* any change in the strength of the current This property of the circuit is called "Inductance" and just as Inertia or Momentum opposes any change in the motion of a body, so also the "Inductance" of an electric circuit opposes any change in the flow of the current through it

12 **Capacity** —In addition to resistance and inductance, an electric circuit may also possess a third property known as "Capacity" It may be due to the presence of Plate condenser effect or it may exist as an electro static capacity between the conductor of a cable and another conductor or earth

"Capacity" can be defined as that property of an electric circuit which has a [illegible] effect on the flow of the current It is due to the fact that when it is present in an A C circuit, some current given by the source of supply will be employed for charging the condenser and that it will receive the charging current at each reversal of the voltage remembering that the quantity of the electricity stored by the condenser is given back to the circuit during the period when the voltage is low It is quite evident that the current charging the condenser will raise its potential and when the applied voltage is low, the condenser will find its potential higher and therefore it will send back the current to the circuit

The capacity plays an important part in A C circuits and the electrical engineer should be able to form a clear mental picture of the nature and characteristics of this property of an alternating current circuit We can best explain it by illustrating a mechanical analogy Imagine there are two tanks A and B connected together by means of a tube MP as shown in Fig 5 The portions MN and OP of the

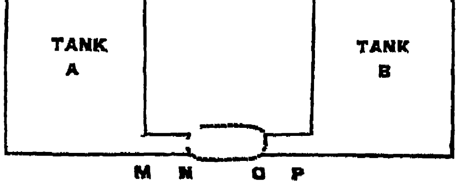


FIG 5

Mechanical Analogy illustrating Capacity

tube are made of cast iron, while the portion NO is made of an elastic rubber Further imagine that the water is being pumped alternately forward and backward between the tanks Now, if the connecting tube is entirely made of metal, the tube will offer only resistance to the flow of water, but it will in no way *modify* the flow But if we substitue the portion NO of the metal tube by an elastic rubber, we can easily see that the rubber will be undergoing periodic expansions and contractions This is due to the fact that when the pressure of water is increasing in the tube, part of water having pressure behind, will stretch the tube and it will continue stretching, taking more and more water all the time until its elasticity counterbalances the pressure of the water near the tube As soon as the pressure of water begins to decrease, the stretched tube will give back the water held in the stretched portions of the tube It is evident, therefore, that if the variations in the pressure of water, flowing through the connecting tube, is periodic there will be a periodic flow of water in and out of the elastic tube An exactly similar phenomenon takes place in an alternating current circuit possessing "Capacity"

13 **Hydraulic Analogy to illustrate Capacity** —The following comparison may further illustrate the effect of 'Capacity" in alternating current circuits

No	Hydraulic	No	Electric
1	The connecting tube made of iron and elastic rubber	1	The circuit possessing ohmic resistance and capacity
2	When the water pressure increases, part of water, having pressure behind, flows into the stretching rubber until its elasticity balances the applied pressure	2	When the alternating voltage increases and the condenser begins to be charged, part of the current supplied by the alternator is stored, in a sense in the condenser until the e m f developed in the condenser, known as the condenser e m f, balances the applied voltage
3	When the pressure of water in the pipe	3	When the applied voltage begins to

begins to decrease, the elasticity of rubber pumps back part of the water into the pipe	decrease, the e m f developed in the condenser sends back part of the current into the circuit
4 If the pressure of water remains constant, the presence of the elastic rubber does not interfere with the flow of water through the pipe, but should the pressure of water vary and vary periodically, the water would flow periodically in and out of the stretched rubber tube	4 If the applied voltage remains constant as would be the case in direct current circuit, the condenser property of the circuit will not interfere with the flow of normal current, but should the voltage be alternating, there would be a charging current at each reversal, and the charge periodically absorbed would be given back during periods of low applied voltage
5 Owing to the presence of elastic rubber, there will be an	5 Owing to the presence of capacity in an alternating current

initial rush of water into the stretched tube in advance of the normal flow of water through the pipe

6 It is therefore clear that the presence of elastic rubber in the pipe *complicates* the flow of water through the pipe, in other words it has a *modifying* influence on the flow of water in the pipe

circuit, there will be an initial rush of current at each reversal of voltage in addition to and in advance of the normal current of the circuit

6 It is therefore clear that the presence of condenser property in an alternating current circuit *complicates* the normal flow of current in the circuit, in other words it has a *modifying* influence on the flow of current in the circuit

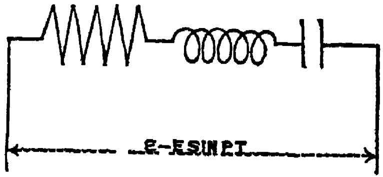
CHAPTER III

PHASE RELATIONSHIP BETWEEN ALTERNATING VOLTAGE AND CURRENT

14 **The Components of Alternating Voltage** We had observed in Chapter I that the current set up in a circuit is constant if the impressed voltage is constant assuming that other factors do not vary, and the current is alternating if the impressed voltage is alternating In Fig 2, the alternating current flowing in a circuit is shown to be in phase with the impressed voltage, but this condition is fulfilled only when the circuit possesses no other property except that of ohmic resistance Should the circuit possess either inductance or capacity or both of these in addition to ohmic resistance, the alternating current flowing through such a circuit might not remain in phase with the impressed voltage In a circuit containing ohmic resistance, inductance, and capacity, all connected in series, the impressed alternating voltage will be the resultant of voltages across resistance, inductance, and capacity We shall now study the phase relationship between

the impressed voltage and the current, and also the phase relationship of the components of the impressed voltage with respect to their resultant and the current. Suppose that an alternating current is flowing through a circuit containing resistance, inductance, and capacity joined in series such as that shown in Fig 6. Let the current be represented by curve No I of Fig 7. It is evident that the impressed voltage driving this current will have three components, namely —

FIG 6



(i) Ohmic Resistance component

(ii) Inductance component

(iii) Capacity component

15 **Ohmic Resistance Component of the Impressed Voltage**—A component of the impressed voltage will be needed to supply a voltage drop across the ohmic resistance. And this drop at any instant will be equal to the current at that instant multiplied by the ohmic resistance. The resistance voltage component of the impressed voltage will, obviously, be in phase with the current flowing through the circuit and therefore will be represented, for

phase relationship, by curve No II of Fig 7

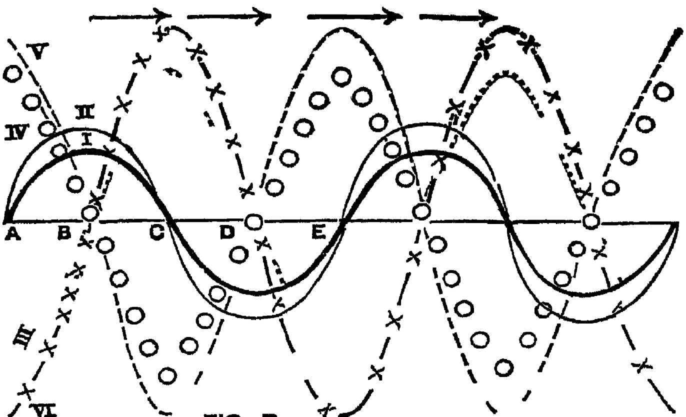


FIG 7

Graphs of Current and Components of Impressed Voltage

I	▬▬	Current
II	——	Resistance Component of Impressed Voltage
III		e m f induced due to Inductance
IV	- O - O	Inductance Component of Impressed Voltage
V	- - - -	e m f developed in Condenser
VI	— × — × —	Capacity component of Impressed Voltage

16 **Inductance Component of the Impressed Voltage**—We know that when a circuit is carrying an alternating current, the magnetic flux associated with the circuit will also be alternating and the varying flux will induce an alternating e m f known as inductance e m f This inductance e m f is in such a direction that it opposes the change in current A glance at the current curve No I, Fig 7, will show that the maximum variation

in the rate of change of current takes place at points A, C and E, *viz* when the current is zero, and therefore at such points the change in flux is greatest with the result that the e m f induced, due to inductance, is maximum It will also be observed that the change of current is minimum at points B and D, *viz* when the current is maximum So the variation in the strength of magnetic flux, due to the current, will be minimum at points B and D and therefore the inductance e m f will be minimum It is therefore clear that the inductance e m f is of maximum value when the current is zero, and it is zero when the current is maximum We may now proceed to investigate the phase position of the inductance e m f curve relative to the current curve Since the e m f induced, due to inductance, is always in such a direction that it opposes the change in current, the inductance e m f will be maximum and negative when the current is zero and is beginning to increase in the positive direction The inductance e m f will be maximum and positive when the current is zero and is going to increase in the negative direction It should also be noted that when the current is positive and decreasing, the inductance e m f will

also be positive since it must be positive to be able to oppose the decrease in the current Similarly when the current is negative and decreasing the inductance e m f must be negative When the current is positive and increasing, the inductance e m f will be negative, and when the current is negative and increasing, the inductance e m f will be positive Therefore if we construct the inductance e m f curve to satisfy the above conditions, we find it will be repressed by curve No III (Fig 7), and that it lags 90 behind the current The impressed voltage will have to provide a component to overcome this inductance e m f This component will be opposed to the inductance e m f and as such will have to be 90 in advance of the current The inductance component of the impressed voltage, therefore, will be represented by curve IV (Fig 7) The difference between the inductance e m f and the inductance component of the impressed voltage should be noted The former is the e m f induced in the circuit due to inductance and the latter is that component of the impressed voltage which overcomes the former and is therefore called the inductance component of the impressed voltage We see, therefore, that the current lags 90

behind the inductance component of the impressed voltage

17 **Capacity Component of the Impressed Voltage**—A condenser present in an alternating current circuit, either in the form of a plate condenser or as an electro static capacity between the conductor and another conductor or earth, becomes the seat of an e m f known as the condenser e m f The e m f developed in the condenser is due to the charging current and this e m f is proportional to the quantity of electricity stored

The investigation of the phase of the capacity component of the impressed voltage becomes easier if we remember the following three principles —

(*i*)—That the condenser e m f, *i e*, the e m f developed in the condenser, due to the charging current, is proportional to the quantity of electricity stored

(*ii*)—When the current is positive, the e m f that is being developed in the condenser will have a polarity such that when the current changes from positive to negative, the condenser will supply back the current in the direction opposite to the direction in which the current was flowing when the condenser e m f was

being developed That is to say, the condenser e m f may be regarded as negative when the charging current is positive and *vice versa*

(*iii*)—Since the condenser e m f is proportional to the quantity of electricity stored, this e m f will rise to a maximum value when the maximum quantity of electricity has been stored, *i e* when the current is changing from positive to negative or negative to positive

By applying the above principles it becomes clear that when the current is changing from positive to negative as at point C, the condenser e m f is maximum and negative, and when the current is changing from negative to positive as at E, the condenser e m f will again be maximum but positive The curve V (Fig 7), shows the phase of the condenser e m f It is 90 in advance of the current To overcome this e m f the impressed voltage will have to provide a component, known as capacity component, equal and opposed to the condenser e m f This component will be represented by curve No VI (Fig 7) The current is therefore leading the capacity component of the impressed voltage by 90

18 **Impressed Voltage**—The resultant of the components discussed above will be the

impressed voltage The three components of the impressed voltage are—

(1) Resistance component in phase with the current (Curve II)

(2) Inductance component 90 in advance of the current (Curve IV)

(3) Capacity component 90 behind the current (Curve VI)

If these three curves representing the three components are combined, it will be found that the impressed voltage will have one of the following phase positions relative to the current —

(*a*) In phase with the current This will be the case when the inductance component and the capacity component are equal in magnitude and therefore the two, being in opposition in regard to phase, will neutralise each other

(*b*) In advance of the current This will happen when the inductance component is bigger in magnitude than the capacity component, and the addition of the two will be a curve which is 90 in advance of the current The combination of this

curve, and the ohmic resistance component curve will give the resultant impressed voltage that will be in advance of the current

(c) Lagging behind the current This will happen when the capacity component is bigger than the inductance component The addition of the two will be a curve which is 90 lagging behind the current The combination of this curve with the resistance component will give the resultant impressed voltage that will be lagging behind the current

It will be a very good practice if the student will draw the current curve and the three components of the impressed voltage, and investigate for himself the phase position of the impressed voltage relative to the current and the voltage components Such an independent investigation will enable him to form a clear mental picture of what is actually happening in a circuit in which alternating current is flowing

CHAPTER IV

INTRODUCTION TO THE SYMBOLIC METHOD

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19 **Determination of Magnitude and Phase Angles of Alternating Currents and Voltages belonging to a Circuit**—It has been shown in the foregoing chapters, that the property of a direct current circuit, that is of any importance, is its ohmic resistance But in an alternating current circuit the other two properties, namely inductance and capacity, also become determining factors of the strength of the current and its phase position with respect to the impressed voltage We had observed in the preceding chapter, how the presence of inductance and capacity introduces more than one component in the impressed voltage and complicates the relative phase positions of currents and voltages belonging to the circuit To illustrate it better, let us take an example of an alternating current circuit possessing ohmic resistance, inductance, and capacity in a manner such as that

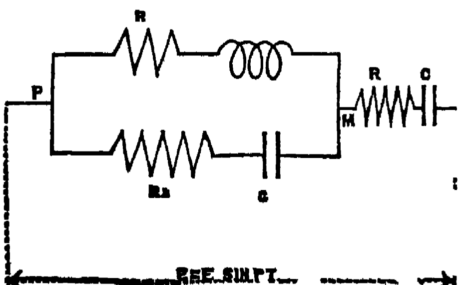


FIG 8

shown in Fig 8 and that a sine wave voltage, expressible by the equation $e = E \sin pt$, is impressed across PN The main circuit in this case consists of two branch circuits PM and MN connected in series The circuit PM has two paths in parallel, one containing resistance and inductance and the other resistance and capacity If the phase relations existing between branch currents, main current, voltages across PM and MN and the impressed voltage across PN are determined, it will be found that

(*a*) the impressed voltage, *i e* the voltage across PN has two components, one across PM and the other across MN The two components may be of different magnitudes and will have certain phase relationship with each other and each may have a different phase relation with the impressed voltage

(*b*) The main current will bear a distinct ~~certain~~ phase relation with the impressed voltage

(*c*) The main current will have two components representing the currents in two branch circuits of PM, different in magnitude and in phase relation with the resultant current, *i e* the main current

It is evident from the above illustration, that the more complicated, in the manner in which the circuit is composed as regards resistance, inductance, and capacity, is the alternating current circuit, the more difficult becomes the study and determination of the mutual phase relations and magnitudes of currents and voltages entering into the problem Hence arises the need of a method that could, without very lengthy calculations and without complicated graphical constructions, be used for determining the exact magnitudes and phase relations of currents and voltages belonging to an alternating current circuit The method should also have the advantage of presenting, at every step, a clear mental picture of the phase positions of currents and voltages

The two well known methods commonly employed are

(a) The *Analytical Method*, and

(b) The *Graphical Method*

20 **The Analytical Method**—The Analytical method is generally used when exact calculations of the magnitudes and phase relations of currents and voltages are necessary, because these are determined trignometrically The method suffers from two disadvantages, namely, that it

does not provide at each step a clear mental picture of the various phase relations, and the mathematics involved in the calculation is, sometimes, very lengthy

21 **The Graphical Method** —This method involves the use of Polar Co ordinates It has the advantage that it presents a clear mental picture of the phase relations of currents and voltages entering into the problem but it suffers from the drawback that it is not suited in cases where exact values of the magnitudes and phase angles of currents and voltages are required and the graphical constructions involved are generally tedious

22 **The Symbolic Method** —There is a third method not so well known as the other two methods It combines in it the exactness of the Analytical Method and the clearness of the Graphical Method It enables the student to determine the exact values of currents voltages, and phase angles without ~~indulging in~~ lengthy mathematics, and also permits him to form a clear mental picture, without tedious graphical constructions, of the phase relationships between currents and voltages entering into a problem

CHAPTER V

THE SYMBOLIC METHOD

23 Standard Practice in Graphical Constructions —The alternating current work involves certain graphical constructions in cases where it is necessary to represent graphically the magnitude and phase position of currents and voltages Since the graphical representation can be made in several ways, it is strongly recommended to make it, for the purpose of clearness and uniformity, a standard practice to observe strictly the following rules —

1 That the positive horizontal direction, as denoted by the line OX, Fig 9, represents the

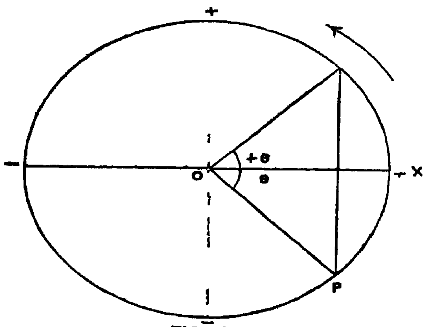


FIG 9

Vector Diagram

Vector of Reference for measurement of phase angles of all vectors For instance a vector having a phase angle equal to zero will lie in the direction OX

2 All angles will be measured from OX, positive angles in counter clockwise direction and negative angles in clockwise direction For instance a vector having a phase angle θ will be shown on a diagram by a line such as OP, Fig 9, and a vector having a phase angle $-\theta$ will be represented by a line such as OP^1

3 That a rotating vector, representing either current or voltage of an alternating current circuit, will move in the positive direction and therefore it will revolve counter clockwise

24 **Study of the Symbolic Method** — We may now proceed with the study of the Symbolic Method We know that an alternating current or voltage sine wave of a definite frequency can be represented graphically, in magnitude as well as in phase, by a vector such as OP, Fig 10

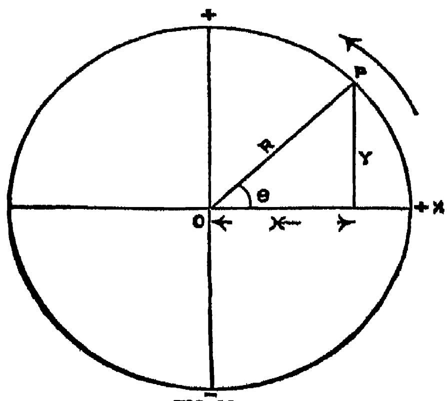


FIG. 10

It can be denoted analytically if we know the two quantities namely its maximum value R and its phase angle θ with respect to our standard *Vector of Reference* The analytical expression would be $i = I \sin(pt + \theta)$ in the case of current sine wave and $e = E \sin(pt + \theta)$ in the case of voltage sine wave where I or E stands for R

If we represent the vector OP by rectangular co ordinates x and y, we can easily determine the magnitude R of the vector and its phase angle θ, as OPM being a right angled triangle,

$$R = \sqrt{x^2 + y^2} \text{ and } \theta = \tan^{-1}\frac{y}{x}$$

The vector OP, which may represent alternating current or alternating voltage or impedance, can be denoted both in magnitude and in phase angle by the following various ways —

(*a*) By showing the magnitude and position of the vector on a vector diagram

(*b*) By representing the vector in polar co ordinates, *i e* by denoting that its magnitude is R and its phase angle is θ

(*c*) By representing the vector in rectangular co ordinates, *i e*, by denoting that its horizontal component is X and its

vertical component is Y These two components will suffice to represent the vector both in magnitude and phase position, because R would be

$$= \sqrt{x^2 + y^2} \text{ and } \theta = \tan^{-1} \frac{y}{x}$$

In addition to the above ways, we can use another way, *viz*, if we denote a vector OP simply by a complex quantity such as $x + jy$ where j is merely a symbolic index employed to distinguish the component y from the component x It signifies that y, which is accompanied by the distinguishing index, is the vertical component, and x, not so accompanied, is the horizontal component and that x and y are to be added geometrically It should be observed that this is really the same method as indicated under (*c*) above with the difference that whereas under (*c*) we have to write the statement that the vector OP, whose magnitude and phase position it is desired to ascertain, is represented by vertical component = y and by horizontal component = x, under the symbolic method we simply write down that the vector OP is represented by $x + jy$

$x + jy$ conveys to us clearly every thing that is indicated in the above lengthy statement and at

the same time permits itself to be employed in calculation work

It is emphasised that when a student comes across such a complex quantity he must interpret it quickly as meaning that it represents a certain vector, both in magnitude and in phase position, because the magnitude $= \sqrt{x^2 + y^2}$ and phase angle is equal to $\theta = \tan^{-1} \frac{y}{x}$

In other words, whenever alternating current, voltage, or impedance vector is denoted by an imaginary complex quantity such as $x + jy$, the student should at once take it as meaning that

(*i*) the vector is represented by rectangular co ordinates, x is the horizontal component and y (as it is accompanied by the symbol j) is the vertical component and

(*ii*) the magnitude of the vector can be represented by the *geometrical* additions of the values x and y (since x and y are at right angles to each other)

Therefore $R = \sqrt{x^2 + y^2}$ and its phase angle $= \theta$, where $\theta = \tan^{-1} \frac{y}{x}$

It should be noted that some Electrical Engineers, in the application of the method, use some other symbolic letter instead of j, the more commonly used is the letter i. It is strongly recommended that the letter i should not be used as it introduces little confusion for i is also employed to represent the instantaneous value of an alternating current. It will be seen later that a symbolic expression such as $x + jy$ representing say impedance of an alternating current circuit can be employed as an operator for the determination of the exact values of the magnitude and phase angle of currents and voltages entering into a problem. At the same time it affords us, without the need of graphical construction, a clear mental picture of how currents and voltages belonging to the circuit stand in regard to phase position.

Let $y = OM$, Fig. 11, represent a vector in the horizontal direction.

Then jy, represented by ON, will represent a vector of magnitude $= y$ but turned through $+90$

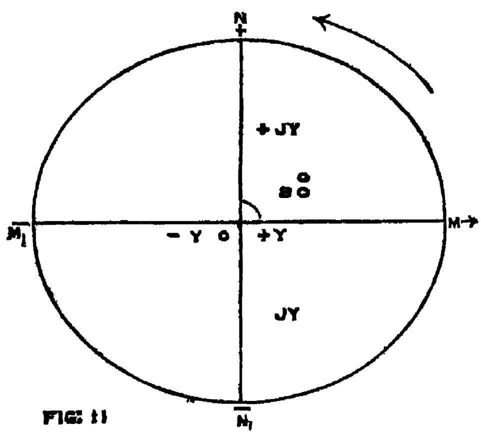


FIG. 11

The distinguishing index j denotes simply the operation of turning the vector through 90 in the positive direction, *i e* in counter clock wise direction measuring the angle from OX which is taken as the standard direction of reference If the vector ON, which is symbolically expressed by jy, is turned through another 90 to the position of OM_1, it will then be denoted by j y where j^2 really means that y has been turned 2×90 *i e* through $+180$ If we turn the vector OM_1, which is represented symbolically by j y, through an angle 90 to the position of ON_1, it will then be denoted by j^3y, meaning that y has been turned through an angle 3×90 *i e* through $+ 270$ If we repeat the same operation and further turn ON_1 through 90 to the position OM, it will be denoted by j^4y meaning that y is to be turned through 4×90 *i e* through $+ 360$

Referring to Fig 11, if $OM = + y$, then $OM_1 = - y$, but we have shown that while OM is represented by either y or j^4y, OM_1 is represented by j y

It is clear therefore that $j^2 = -1$ and $j^4 = +1$ giving $j = \sqrt{-1}$

Again ON was represented by jy and ON_1 by

$j^3 y$ or $-jy$, therefore we have $j^3 = -j$ or $j^2 = -1$ or $j = \sqrt{-1}$

It is to be noted that although we had adopted the letter j as a mere distinguishing index, indicating that when it accompanies any real quantity it simply denotes that the quantity has been turned through $+90$ (measuring the angle from our standard direction of reference), it can also be made, as we have observed above, to stand for $\sqrt{-1}$

It has been stated above that $x + jy$, representing a vector, denotes that its horizontal component is $+x$ and its vertical component is $+y$, and therefore the magnitude of the vector will be $\sqrt{x^2 + y}$ and its phase angle $= \theta$

where $\theta = \tan^{-1} \dfrac{y}{x}$

Similarly a vector expressible by say

$-x - jy$ will have its magnitude

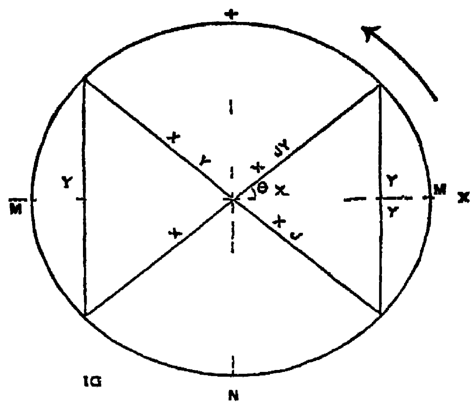
$= \sqrt{(-x)^2 + (-y)^2}$, *i e* $\sqrt{x^2 + y}$

and its phase angle $= \theta$

where $\theta = \tan^{-1} \dfrac{-y}{-x} = \tan^{-1} \dfrac{y}{x}$

This vector has its horizontal component equal to $-x$ *i e* in the direction OM_1 and its

vertical component equal to $-y$ i e in the direction ON_1 (Fig 12)



It should be observed that vectors $x + jy$ and $-x - jy$ are equal in magnitude but differ in phase position by an angle = 180 The former has a phase angle $= \tan^{-1}\frac{y}{x}$ with respect to the vector of reference OX and the latter has a phase angle $= \tan^{-1}\frac{y}{x} + 180$ with respect to the vector of reference The reason being that the vector $-x - jy$ has its components $-x$ and $-y$ and therefore it lies in the third quadrant The student should therefore be careful in interpreting the meaning of + and − signs For instance, we have shown on the last page that the phase angles of both

vectors $x + jy$ and $-x - jy$ are equal to $\tan^{-1}\frac{y}{x}$ In the first case the angle is measured from OX in the second case the angle is $\tan^{-1}\frac{y}{x} + 180$, since the vector lies in the third quadrant

Again, a vector represented by $x - jy$ will have its magnitude $= \sqrt{x^2 + (-y)^2}$ i e $= \sqrt{x^2 + y^2}$ and its phase angle $= \tan^{-1}\frac{-y}{x} = -\tan^{-1}\frac{y}{x}$ The vector therefore lies in the fourth quadrant because x is positive and y is negative The vector $-x + jy$ will have its magnitude $= \sqrt{(-x)^2 + (+y)^2}$ i e $= \sqrt{x^2 + y^2}$ and its phase angle $= \tan^{-1}\frac{y}{-x}$

$$= -\tan^{-1}\frac{y}{x}$$

It should be carefully noted that the vector $-x + jy$ lies in the second quadrant and although the angle is $-\tan^{-1}\frac{y}{x}$,

it is really $-\tan^{-1}\frac{y}{x} - 180$

The student should make himself perfectly clear as to how $x + jy$, $-x - jy$, $x - jy$ and $-x + jy$ represent vectors same

in magnitude but differing in phase positions with respect to the vector of reference OX as shown in Fig 12

25 **To Multiply a vector by j**—It has been shown in the foregoing discussion that multiplying a vector by the symbolic index j means that the magnitude of the vector is to be increased in the ratio of 1 1 and that the vector is to be turned through an angle of $+90$

26 **To Multiply a vector by $-j$**—Similarly a vector when multiplied by $-j$ has its magnitude increased in the ratio of 1 1 and has itself turned through an angle of -90

27 **To Multiply a Vector R by a Vector $x+jy$**—If we multiply R by $x+jy$, we get a new vector denoted by

$$R \times (x+jy)$$

$$\text{or } Rx+jRy \qquad (1)$$

Now $Rx+jRy$ represents a vector whose magnitude is equal to R_1, where $R_1 = \sqrt{R^2x^2+R^2y^2} = R\sqrt{x^2+y^2}$ and whose phase angle $= \theta_1$, where

$$\theta_1 = \tan^{-1}\frac{Ry}{Rx} = \tan^{-1}\frac{y}{x}$$

The new vector is therefore of magnitude $R\sqrt{x^2+y^2}$ and is at an angle $\theta_1 = \tan^{-1}\frac{y}{x}$ from the normal position of R

Hence, multiplying a vector R by $x + jy$ means that the magnitude of the vector R is to be increased in the ratio $1 : \sqrt{x + y^2}$ and that the vector is to be turned through an angle $+ \tan^{-1} \frac{y}{x}$

28 **To Multiply a vector R by a vector x − jy**.—If we multiply R by $x - jy$, we get a new vector represented by

$$R \times (x - jy)$$

$$\text{or } Rx - jRy \qquad 2$$

Now $Rx - jRy$ represents a vector whose magnitude is equal to R_1, where

$$R_1 = \sqrt{R^2x^2 + (-Ry)^2}$$

$$= R \sqrt{x^2 + y^2}$$

and whose phase angle $= \theta_1$, where

$\theta_1 = \tan^{-1} \frac{-Ry}{Rx} = -\tan^{-1} \frac{y}{x}$ The new vector is therefore of magnitude $R \sqrt{x^2 + y^2}$ and is at an angle $\theta_1 = -\tan^{-1} \frac{y}{x}$ from the original position of R

Hence, multiplying a vector R by $x - jy$ means that the magnitude of R is to be increased in the ratio $1 : \sqrt{x^2 + y^2}$ and that the vector is to be turned through an angle $= -\tan^{-1} \frac{y}{x}$

29 **To divide a vector R by a vector $x + jy$**

When a vector say R is divided by a vector $x + jy$ the new vector is represented by

$$\frac{R}{x + jy}$$

$$= \frac{R\,(x - jy)}{(x + jy)\,(x - jy)} = \frac{Rx - jRy}{x^2 - j^2\,y^2}$$

$$= \frac{Rx - jRy}{x^2 + y^2}, \text{ since } j = -1,$$

$$= \frac{Rx}{x^2 + y^2} - j\,\frac{Ry}{x^2 + y^2}$$

The new vector will therefore be denoted by

$$\frac{Rx}{x + y^2} - j\,\frac{Ry}{x^2 + y^2} \qquad 3$$

The magnitude of the new vector therefore will be R_1,

$$\text{where } R_1 = \sqrt{\frac{R\ x^2}{(x^2 + y^2)^2} + \frac{R^2\ y^2}{(x^2 + y^2)^2}}$$

$$= \sqrt{\frac{R^2\,(x^2 + y^2)}{(x^2 + y^2)^2}}$$

$$= \frac{R}{\sqrt{x^2 + y^2}}$$

and the phase angle of the new vector will be θ_1,

$$\text{where } \theta_1 = \tan^{-1} \frac{-\dfrac{Ry}{x^2 + y^2}}{\dfrac{Rx}{x^2 + y^2}} = \tan^{-1} \frac{-Ry}{Rx}$$

$$= -\tan^{-1} \frac{y}{x}$$

Therefore, dividing a vector R by a vector $x + jy$ means dividing the magnitude of the vector R by the magnitude of the vector $x + jy$ i e by $\sqrt{x^2 + y^2}$ and turning the vector R through an angle $-\tan^{-1}\frac{y}{x}$

30 **To Divide a Vector R by a Vector $x - jy$** When a vector R is divided by a vector $x - jy$, the result is denoted by $\frac{R}{x - jy}$

$$= \frac{R\,(x + jy)}{(x - jy)\,(x + jy)}$$

$$= \frac{Rx + jRy}{x^2 - j^2 y^2} = \frac{Rx + jRy}{x^2 + y^2}$$

$$= \frac{Rx}{x^2 + y^2} + j\frac{Ry}{x^2 + y^2} \qquad (4)$$

The new vector will therefore have a magnitude $= R_1$, where $R_1 = \sqrt{\frac{R^2 x^2}{(x^2 + y^2)^2} + \frac{R^2 y^2}{(x^2 + y^2)^2}}$

$= \sqrt{\frac{R^2\,(x^2 + y^2)}{(x^2 + y^2)^2}} = \frac{R}{\sqrt{x^2 + y^2}}$ and its phase angle will be θ_1, where $\theta_1 = \tan^{-1}\frac{\frac{Ry}{x^2 + y^2}}{\frac{Rx}{x^2 + y^2}} = \tan^{-1}\frac{y}{x}$

Hence, dividing a vector R by a vector $x - jy$ means dividing the magnitude R by the magnitude of the vector $x - jy$, i e by $\sqrt{x^2 + y^2}$ and turning

the vector R through an angle $= -\tan^{-1}\frac{-y}{x}$

$= +\tan^{-1}\frac{y}{x}$

31 **To Multiply a Vector R by a Vector**

$$\frac{(x_1 + jy_1)(x + jy_2)(x_3 + jy_3)}{(a_1 + jb_1)(a_2 + jb_2)(a_3 + jb_3)}$$

Let us take a general case where a vector say R is multiplied by a vector

$$\frac{(x_1 + jy_1)(x + jy)(x_3 + jy_3)}{(a_1 + jb_1)(a + jb)(a_3 + jb_3)}$$

We know that vector

$x_1 + jy_1$ is of magnitude $\sqrt{x_1^2 + y_1^2}$ and has a phase angle $\theta_1 = \tan^{-1}\frac{y_1}{x_1}$,

$x_2 + jy_2$ „ „ $\sqrt{x_2^2 + y_2^2}$ and has a phase angle $\theta_2 = \tan^{-1}\frac{y_2}{x_2}$,

$x_3 + jy_3$ „ „ $\sqrt{x_3 + y_3^2}$ and has a phase angle $\theta_3 = \tan^{-1}\frac{y}{x_3}$

Similarly

$a_1 + jb_1$ „ „ $\sqrt{a_1^2 + b_1^2}$ and has a phase angle $\alpha_1 = \tan^{-1}\frac{b_1}{a_1}$,

$a_2 + jb_2$ „ „ $\sqrt{a_2 + b_2}$ and has a phase angle $\alpha_2 = \tan^{-1}\frac{b_2}{a_2}$,

$a_3 + jb_3$ „ „ $\sqrt{a_3^2 + b_3}$ and has a phase angle $\alpha_3 = \tan^{-1}\frac{b_3}{a_3}$

By applying the rules established in the foregoing sections, the result of multiplication of vector R by $(x_1 + jy_1)\ (x_2 + jy_2)\ (x_3 + jy_3)$ will be to multiply the magnitude R by $\sqrt{x_1^2 + y_1^2} \times \sqrt{x_2 + y_2^2} \times \sqrt{x_3 + y_3^2}$ and to turn the vector through the angle equal to

$$+ \tan^{-1}\frac{y_1}{x_1} + \tan^{-1}\frac{y}{x} + \tan^{-1}\frac{y_3}{x_3},$$

and similarly the result of dividing the vector R by a vector $(a_1 + jb_1)\ (a \ + jb_2)\ (a_3 + jb_3)$ will be to divide the magnitude R by $\sqrt{a_1 + b_1} \times \sqrt{a \ + b} \times \sqrt{a_3^2 + b_3^2}$ and to turn the vector R through the angle equal to

$$-\tan^{-1}\frac{b_1}{a_1} - \tan^{-1}\frac{b_2}{a} - \tan^{-1}\frac{b_3}{a_3}$$

Therefore, the net result of multiplying a vector R by a symbolic expression—

$$\frac{(x_1 + jy_1)\ (x \ + jy \ )\ (x_3 + jy_3)}{(a_1 + jb_1)\ (a \ + jb_2)\ (a_3 + jb_3)}$$

is to multiply the magnitude R by

$$\frac{\sqrt{x_1 + y_1} \times \sqrt{x_2 + y_2} \times \sqrt{x_3 + y_3}}{\sqrt{a_1 + b_1^2} \times \sqrt{a_2 + b} \times \sqrt{a_3 + b_3^2}}$$

and to turn the vector R from its normal phase position through an angle equal to

$$+\tan^{-1}\frac{y_1}{x}+\tan^{-1}\frac{y}{x}+\tan^{-1}\frac{y_3}{x_3}-\tan^{-1}\frac{b_1}{a_1}-\tan^{-1}\frac{b}{a}$$

$$-\tan^{-1}\frac{b_3}{a_3}$$

The above result can also be arrived at in the following manner —

Let the vector $x_1 + jy_1$ be of magnitude R_1 making an angle θ_1 with the standard vector of reference, which we have assumed to lie in the positive horizontal direction Therefore,

$$R_1 = \sqrt{x_1^2 + y_1^2}\text{, and } \theta_1 = \tan^{-1}\frac{y_1}{x_1}$$

Hence, $x_1 = R_1 \cos\theta_1$, and $y_1 = R_1 \sin\theta_1$

Similarly, if the vector $x_2 + jy$ is of magnitude R_2 and makes an angle θ with the standard vector of reference, we have

$$R_2 = \sqrt{x_2^2 + y_2^2}\text{, and } \theta_2 = \tan^{-1}\frac{y}{x_2}$$

Hence, $x_2 = R_2 \cos\theta$, and $y = R_2 \sin\theta_2$

Similarly, it can be shown that for a vector of magnitude R_3 and phase angle θ_3 and expressible by $x_3 + jy_3$, we shall have

$$x_3 = R_3 \cos\theta_3\text{, and } y_3 = R_3 \sin\theta_3$$

Also if S_1, S_2, and S_3 are the respective magnitudes and α_1, α_2, and α_3 the corresponding phase angles of vectors represented symbolically by $a_1 + jb_1$, $a_2 + jb_2$, and $a_3 + jb_3$, it can be shown that

$$a_1 = S_1 \cos \alpha_1 \quad \text{and} \quad b_1 = S_1 \sin \alpha_1$$

and $a_2 = S_2 \cos \alpha_2$ and $b_2 = S_2 \sin \alpha_2$

also, $a_3 = S_3 \cos \alpha_3$ and $b_3 = S_3 \sin \alpha_3$

Therefore, the expression

$$R \times \frac{(x_1 + jy_1)\,(x_2 + jy_2)\,(x_3 + jy_3)}{(a_1 + jb_1)\,(a_2 + jb_2)\,(a_3 + jb_3)} \quad \text{becomes}$$

$$R \times \frac{(R_1 \cos \theta_1 + jR_1 \sin \theta_1)\,(R_2 \cos \theta_2 + jR_2 \sin \theta_2)}{(S_1 \cos \alpha_1 + jS_1 \sin \alpha_1)\,(S_2 \cos \alpha_2 + jS_2 \sin \alpha_2)} \; \frac{(R_3 \cos \theta_3 + jR_3 \sin \theta_3)}{(S_3 \cos \alpha_3 + jS_3 \sin \alpha_3)}$$

$$= R \times \frac{[R_1 (\cos \theta_1 + j \sin \theta_1)]\,[R_2 (\cos \theta_2 + j \sin \theta_2)]}{[S_1 (\cos \alpha_1 + j \sin \alpha_1)]\,[S_2 (\cos \alpha_2 + j \sin \alpha_2)]} \; \frac{[R_3 (\cos \theta_3 + j \sin \theta_3)]}{[S_3 (\cos \alpha_3 + j \sin \alpha_3)]}$$

$$= R \times \frac{R_1 e^{j\theta_1} \times R_2 e^{j\theta_2} \times R_3 e^{j\theta_3}}{S_1 e^{j\alpha_1} \times S_2 e^{j\alpha_2} \times S_3 e^{j\alpha_3}}$$

$$= R \times \frac{R_1 R_2 R_3}{S_1 S_2 S_3} \times \frac{e^{j\theta_1} \times e^{j\theta_2} \times e^{j\theta_3}}{e^{j\alpha_1} \times e^{j\alpha_2} \times e^{j\alpha_3}}$$

$$= R \times \frac{R_1 R_2 R_3}{S_1 S_2 S_3} \times e^{j(\theta_1 + \theta_2 + \theta_3 - \alpha_1 - \alpha_2 - \alpha_3)}$$

$$= R \times \frac{R_1 R_2 R_3}{S_1 S_2 S_3} \Big[\cos(\theta_1 + \theta_2 + \theta_3 - \alpha_1 - \alpha_2 - \alpha_3) + j \sin(\theta_1 + \theta_2 + \theta_3 - \alpha_1 - \alpha_2 - \alpha_3)\Big] \qquad (1)$$

Let P stand for $R \times \frac{R_1 R R_3}{S_1 S_2 S_3}$, and ϕ for $(\theta_1 + \theta_2 + \theta_3 \quad - \alpha_1 - \alpha - \alpha_3)$, the expression (1) will become

$$P \cos \phi + jP \sin \phi \qquad (2)$$

We can see that expression (2) represents a vector whose magnitude is—

$$\sqrt{P^2 \cos \phi + P \sin^2 \phi}$$
$$= P \sqrt{\cos^2 \phi + \sin \phi} = P,$$

and whose phase angle is $\tan^{-1} \frac{P \sin \phi}{P \cos \phi}$

$$= \tan^{-1} \tan \phi = \phi$$

We note, therefore, that the magnitude of the new vector is equal to P,

where $P = R \times \frac{R_1 \times R_2 \times R_3}{S_1 \times S_2 \times S_3}$

$$= R \times \frac{\sqrt{x_1^2 + y_1^2} \times \sqrt{x^2 + y} \times \sqrt{x_3^2 + y_3}}{\sqrt{a_1 + b_1^2} \times \sqrt{a^2 + b_2^2} \times \sqrt{a_3^2 + b_3}},$$

and its phase position is given by an angle ϕ, where $\phi = (\theta_1 + \theta_2 + \theta_3 +$

$$- \alpha_1 - \alpha_2 - \alpha_3)$$

$$= (+\tan^{-1} \frac{y_1}{x} + \tan^{-1} \frac{y_2}{x} + \tan^{-1} \frac{y_3}{x_3}$$

$$- \tan^{-1} \frac{b_1}{a_1} - \tan^{-1} \frac{b_2}{a_2} - \tan^{-1} \frac{b_3}{a_3})$$

32 Imporant Rules of the Symbolic Method—The results obtained in the foregoing discussion are of great importance on account of their application to alternating current circuits The results may therefore be theorized as follows —

(1) The symbol denoted by letter j is used merely as a distinguishing index It denotes that the quantity to which it is attached has been turned through $+90$ if the sign of j is $+$, and through -90 if the sign of j is $-$ The index also equals $\sqrt{-1}$ Therefore j will equal -1, and $j^3 = -j$

(2) A symbolic expression $x + jy$ represents a vector whose rectangular co ordinates are x and y, and therefore the magnitude of the vector is $\sqrt{x^2 + y^2}$

and phase angle $= \tan^{-1} \frac{y}{x}$

(3) A vector represented only by x can be symbolically expressed by ($x + j$ zero), because ($x + j$ zero) denotes a vector whose magnitude is $\sqrt{x^2 + \text{zero}^2} = x$, and

phase angle $= \tan^{-1} \frac{\text{zero}}{x} =$ zero

(4) A vector represented by the symbolic expression $x - jy$ denotes that the magnitude of the vector is $\sqrt{x^2 + (-y)^2} = \sqrt{x^2 + y^2}$, and its phase angle

$$= \tan^{-1} \frac{-y}{x} = -\tan^{-1} \frac{y}{x}$$

The sign $-$ indicates that the angle $\tan^{-1} \frac{y}{x}$ is to be measured in the negative direction, *i e* in the clock wise direction from the direction of the standard vector of reference

(5) A vector represented by the symbolic expression $-x + jy$ denotes that the magnitude of the vector is equal to $\sqrt{(-x)^2 + (y)^2} = \sqrt{x^2 + y^2}$, and its phase angle is equal to $\tan^{-1} \frac{y}{-x}$

$$= -\tan^{-1} \frac{y}{x},$$

measured in the negative direction from the negative horizontal direc tion so that the vector should lie in the second quadrant The student should satisfy himself that the new vector would lie in the second quadrant

(6) A vector represented by the symbolic

expression $-x - jy$ would have its magnitude equal to $\sqrt{-(x)^2+(-y)^2}$ $=\sqrt{x+y^2}$, and its phase angle $=\tan^{-1}\frac{-y}{-x}=\tan^{-1}\frac{y}{x}$ measured in the positive direction from the negative horizontal direction so that the vector should lie in the third quadrant

(7) Multiplying a vector R by a vector denoted by the symbolic expression say $x + jy$ means multiplying the magnitude R by $\sqrt{x^2+y^2}$ and turning the vector R through an angle $= + \tan^{-1}\frac{y}{x}$

(8) Multiplying a vector $x + jy$ by $x_1 + jy_1$ means multiplying a vector 1 by vector $x + jy$ and again by $x_1 + jy_1$, which will involve similar operation as in (7) above, except that in this case the vector to be multiplied is represented by 1 instead of by R and that it is to be multiplied first by $x + jy$ and again by $x_1 + jy_1$ The operations would result in a new vector whose magnitude would equal $\sqrt{x^2+y^2} \times \sqrt{x_1^2+y_1^2}$ and whose

phase angle would be

$+\tan^{-1}\frac{y}{x} + \tan^{-1}\frac{y_1}{x_1}$ measured from the standard vector of reference

It should be noted that when we say vector R or vector 1, we really mean that symbolically it will be denoted by $R + j$ zero and $1 + j$ zero respectively

(9) Multiplying a vector R by a vector $(a_1 - jb_1)$ means that the magnitude R is to be multiplied by $\sqrt{a_1^2 + b_1}$ and the vector R is to be turned through an angle $= +\tan^{-1}\frac{-b_1}{a_1}, = -\tan^{-1}\frac{b_1}{a_1}$

(10) Dividing a vector R by a vector $a + jb$ means dividing the magnitude R by $\sqrt{a^2 + b^2}$ and turning the vector through an angle$= -\tan^{-1}\frac{b}{a}$

(11) Dividing a vector R by a vector $a - jb$ means dividing the magnitude R by $\sqrt{a^2 + b^2}$ and turning the vector through an angle$= -\tan^{-1}\frac{-b}{a} = +\tan^{-1}\frac{b}{a}$

(12) Multiplying a vector R by a vector

$$\frac{(x_1 + jy_1)(x_2 + jy_2)(x_3 + y_3)}{(a_1 + jb_1)(a_2 + jb_2)(a_3 + jb_3)}$$

means that the magnitude R is to be multiplied by

$$=\frac{\sqrt{x_1^2+y_1^2}\times\sqrt{x^2+y_2^2}\times\sqrt{x_3+y_3^2}}{\sqrt{a_1+b_1^2}\times\sqrt{a_2+b_2^2}\times\sqrt{a_3^2+b_3^2}}$$

and that the vector is to be turned through an angle $=\left[+\tan^{-1}\frac{y_1}{x_1}+\tan^{-1}\frac{y}{x_2}+\tan^{-1}\frac{y_3}{x_3}-\tan^{-1}\frac{b_1}{a_1}\right.$ $\left.-\tan^{-1}\frac{b_2}{a}-\tan^{-1}\frac{b_3}{a_3}\right]$

CHAPTER VI

APPLICATION OF THE SYMBOLIC METHOD TO ALTERNATING CURRENT CIRCUITS

33 **Equations of Alternating Current and Voltage Sine Waves**—An alternating current sine wave of maximum value I may be represented on a vector diagram by a vector of constant magnitude OP, (Fig 13), = I revolving

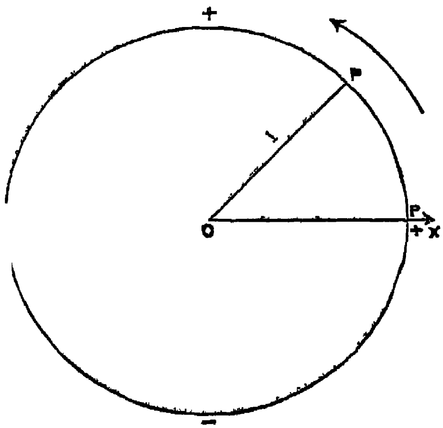


FIG 13

at an angular velocity of p radians per second about O as centre Suppose that at time $t=o$, the vector OP was in the direction OX, *i e* in the direction which we have assumed to represent the standard direction of reference After t seconds the vector will have turned through an angle pt radians Its projection, vertically, at any instant will be I sin pt If

i represents the instantaneous value of the alternating current sine wave we shall have

$$i = I \sin pt \qquad (1)$$

Therefore, we can always represent an alternating current sine wave, analytically, by an equation such as

$$i = I \sin pt, \qquad \text{where}$$

i is the instantaneous value, I the maximum value of the current wave, and p is the angular velocity, in radians per second, of the rotating vector

Now, if T is the time in seconds taken by the vector OP to make a complete revolution, *i e*, to turn through an angle 2π radians, or in other words if T is the time in seconds taken by the wave to pass through one complete cycle, and if f = frequency (number of complete cycles) per second, then clearly

$$T = \frac{1}{f}, \text{ and since } pT = 2\pi,$$

we have $p = \dfrac{2\pi}{T} = 2\pi - 1/f = 2\pi f$

Therefore, equation (1) becomes

$$i = I \sin 2\pi f t,$$

but it is commonly written $i = \sin pt$ where $p = 2\pi f$

Similarly, we can show that the alternating voltage sine wave can be represented by an

equation such as

$$e = E \sin pt$$
$$= E \sin 2\pi f t \qquad (2),$$

where e and E are the instantaneous and the maximum value of the voltage wave

34 **Phase Difference Between Two Alternating Current Sine Waves**—Suppose, we have two alternating current waves expressible, analytically, by the equations

$$i_1 = I_1 \sin pt \qquad (1)$$
$$\text{and } i = I_2 \sin (pt+\theta) \qquad (2)$$

We can see that the alternating current of equation (1) differs from the alternating current of equation (2) not only in magnitude but also in phase The maximum value of current of equation (1) is I_1, and its phase angle with respect to our standard vector of reference (*i e*, the positive horizontal direction) is zero The current represented by equation (2) has a maximum value I_2 and phase angle $= +\theta$ *i e*, the current of equation (2) leads the current of equation (1) by an angle θ It will be noted that both currents have the same angular velocity of p radians per second, and therefore both will revolve with equal angular velocity

It should be clearly understood that

at time $t =$ zero,

$$i = I \sin pt$$

will be represented vectorially by a vector lying in the direction of our standard vector of reference Therefore, a vector represented by the equation of the form $i = I \sin pt$ or $e = E \sin pt$ is always taken as the standard vector of reference for the measurement of the phase angles of all other currents and voltages

When the equations (1) and (2) are represented vectorially, the two vectors will be separated by an angle θ, since one current is leading the other by this angle

We can represent the two currents, both in magnitude and in phase, on a vector diagram such as that shown in Fig 14 The vector OP_1

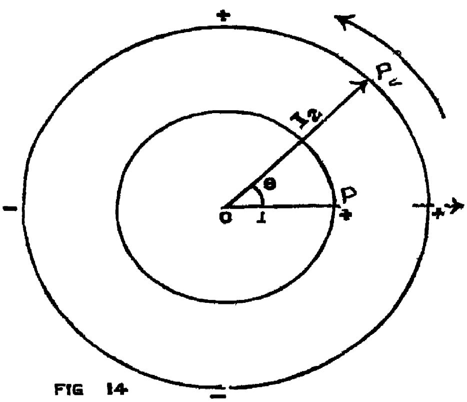


FIG 14

represents the current $i_1 = I_1 \sin pt$, and the vector OP_2 will represent the current $i_2 = I_2 \sin (pt + \theta)$ The two currents can be

represented graphically as shown in Fig 15 One wave represents the current $i_1 = I_1 \sin pt$, and the other wave represents the current $i_2 = I_2 \sin (pt + \theta)$ The current $i_1 = I_1 \sin pt$ is shown lagging behind the current $i_2 = I_2 \sin (pt + \theta)$ by an angle θ

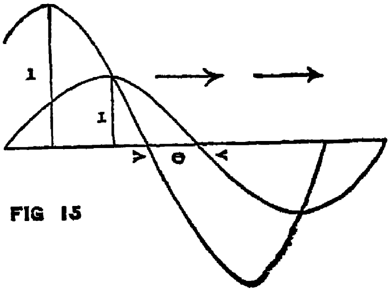


FIG 15

35 **Current and Voltage Sine Waves in Circuit containing Resistance only**—Let us consider a circuit which possesses ohmic resistance only, and in which an alternating current

$$i = I \sin pt \tag{1}$$

is flowing We have to find the magnitude of the impressed voltage driving this current and also the phase angle of the voltage with respect to the current

We know that the impressed alternating voltage required to drive the current in a circuit containing ohmic resistance only is used up entirely in providing the ohmic drop There fore, the voltage across the circuit at any instant will be the product of the ohmic resistance of the circuit and the current flowing through the circuit at that instant

Therefore if resistance = R ohms,

$$e = R \times i \tag{2},$$

can at once find out the current flowing through the circuit by dividing the voltage vector by the impedance vector The latter is to be represented symbolically so that the operation may be done quickly For instance, in a circuit containing resistance = R ohms and impressed with a voltage e = E sin pt, the current flowing through the circuit will be

$$i = \frac{e}{\text{Impedance}} = \frac{E \sin pt}{R + j \text{ zero}}$$

$$= \frac{E}{\sqrt{R^2 + \text{zero}}} \sin \left\{ pt - \tan^{-1} \frac{\text{zero}}{R} \right\}$$

$$= \frac{E}{R} \sin \left(pt - \tan^{-1} \text{zero} \right)$$

$$= \frac{E}{R} \sin \left(pt - \text{zero} \right)$$

$$= \frac{E}{R} \sin pt$$

$$= I \sin pt, \text{ where } I = \frac{E}{R}$$

In both cases discussed above, it will be noted that current and voltage are in phase with each other, which establishes the fact that in a circuit possessing only resistance alternating current is *always* in phase with the impressed voltage

36 Current and Voltage Sine Waves in Circuit containing Resistance and Inductance

A Consider next a circuit such as that shown in Fig 16, having Resistance = R ohms and Inductance = L henries connected in series Let the current flowing in the circuit be

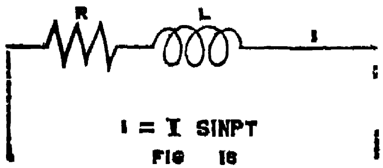


FIG 16

$$i = I \sin pt \qquad (1)$$

We have to find the magnitude and phase angle of the impressed voltage and also of its components across the resistance and the inductance of the circuit We know that the impressed voltage at the instant when the current in the circuit is i will be made up of two components, namely

(i) a component providing the ohmic resistance drop It will be equal to

$$e = Ri$$
$$= R I \sin pt \qquad (2i)$$

and will be in phase with the current

(ii) a component providing the inductive drop It will be equal to

$e_L = L\dfrac{di}{dt}$, which is 90 in advance

of the current, but

$$L\frac{di}{dt} = L\frac{d\ I \sin pt}{dt} = p\ L\ I \cos pt$$

$$= p\ L\ I \sin (pt + 90^\circ)$$

Therefore, the component providing the inductive drop is equal to

$$e_L = p\ L\ I \sin (pt + 90^\circ) \qquad (2a)$$

We note that the maximum value of the resistance component of the impressed voltage is R I, and the maximum value of the inductance component is p L I Since the latter is in advance of the former by 90°, we can add the two symbolically and get the resultant impressed voltage as follows —

The resultant impressed voltage at any instant =

$$e_{impressed} = e + j\,e_L = R\ I \sin pt + j\,p\ L\ I \sin pt$$

$$= (R + j\,p\,L)\ I \sin pt \qquad (3)$$

Multiplying I sin pt by the symbolic expression R + $j\,p$ L, we shall get

$$e_{impressed} = (R + j\,p\,L)\ I \sin pt$$

$$= \sqrt{R^2 + p^2 L^2}\ I \sin \left(pt + \tan^{-1}\frac{pL}{R}\right) \qquad (4)$$

which gives both magnitude and phase angle of the impressed voltage

The student will note that if a circuit contains only resistance and inductance joined

in series, the current vector can be converted into a voltage vector by multiplying the former by the symbolically expressed impedance of the circuit

For instance, if the current flowing through a circuit is known and is expressible by

$$i_1 = I_1 \sin pt \tag{5}$$

and the impedance of the circuit is made up of resistance $= R_1$ ohms and inductance $= L_1$ henries joined in series, we can apply straightaway the symbolic method to determine the magnitude and phase angle of the impressed voltage and its components as follows —

(a) *Magnitude and phase angle of the impressed voltage*

The current is given by equation (5)

The impedance of the circuit

$$= R_1 + j\, p\, L_1 \tag{6}$$

Therefore, the impressed voltage at any instant is equal to

$$\begin{aligned} e &= (R_1 + j\, p\, L_1) \times i_1 \\ &= (R_1 + j\, p\, L_1) \times I_1 \sin pt \\ &= \sqrt{R_1^2 + p^2 L_1^2} \\ &\quad \times I_1 \sin \left(pt + \tan^{-1} \frac{pL_1}{R_1} \right) \end{aligned} \tag{7}$$

which gives both magnitude and phase angle of the impressed voltage

(*b*) *Magnitude and phase angle of the resistance component of the impressed voltage*

The current is given by equation (5)

The impedance of the resistance portion of the circuit $= R_1 + j$ zero (8)

Therefore, the resistance component of the impressed voltage is

$$e = (R_1 + j \text{ zero}) \times i_1$$

$$= (R_1 + j \text{ zero}) I_1 \sin pt =$$

$$\sqrt{(R_1^2 + \text{zero}^2)}\, I_1 \sin\left(pt + \tan^{-1}\frac{\text{zero}}{R_1}\right)$$

$$= R_1 I_1 \sin(pt + \tan^{-1}\text{zero})$$

$$= R_1 I_1 \sin pt \tag{9}$$

which gives both magnitude and phase angle of the resistance component of the impressed voltage

(*c*) *Magnitude and phase angle of the inductance component of the impressed voltage*

The current is given by equation (5)

The impedance of the inductance portion of the circuit $=$ zero $+ j\, p\, L_1$ (10)

Therefore, the inductance component of the impressed voltage is

$$e_L = (\text{zero} + j\, p\, L_1) \times i_1$$

$$= (\text{zero} + j\, p\, L_1) I_1 \sin pt$$

$$= \sqrt{\text{zero}^2 + p^2 L_1^2}\; I_1 \sin\left(pt + \tan^{-1}\frac{pL_1}{\text{zero}}\right)$$

$$= p\, L_1\, I_1 \sin\left(pt + \tan^{-1} \infty\right)$$

$$= p\, L_1\, I_1 \sin\left(pt + 90\right) \qquad (11)$$

which gives both magnitude and phase angle of the inductance component of the impressed voltage

The analytical equations (7), (9), and (11) give both magnitude and phase angle of the impressed voltage and its components

It is left for the student as an exercise to draw the current and the three voltages on a vector diagram, and study how the currrent and voltages are in mutual phase relationship in a circuit containing only resistance and inductance joined in series

The student is strongly recommended to take some ~~little~~ pains in drawing the above results vectorially If he will do that, he will succeed in forming a clear mental picture of how voltages and current are placed, with respect to phase relationship, in a circuit containing resistance and inductance It will also enable him to interpret correctly as to why in actual practice an induction motor, running off an alternating supply, draws the current which lags behind the voltage and therefore the killowatt rating of its stator is not the same as its kilovolt ampere (K V A) rating

$$i = \frac{\text{impressed voltage}}{\text{impedance}}$$

$$= \frac{e}{R + j\,p\,L} = \frac{E \sin pt}{R + j\,p\,L}$$

Dividing, according to the rules established in the preceding chapter, the vector $E \sin pt$ by the symbolic expression $R + j\,p\,L$, we get

$$i = \frac{E \sin pt}{R + j\,p\,L}$$

$$= \frac{E}{\sqrt{R^2 + p^2\,L^2}} \sin\left(pt - \tan^{-1}\frac{p\,L}{R}\right)$$

$$= I \sin\left(pt - \tan^{-1}\frac{p\,L}{R}\right) \qquad (13)$$

where $I = E \div \sqrt{R^2 + p^2\,L^2}$

The equation (13) gives the magnitude of the current and also its phase position with respect to the impressed voltage

(*b*) The resistance component of the impressed voltage will be found as follows —

The current is given by equation (13) The impedance vector of the resistance portion of the circuit = $R + j$ zero

Therefore the voltage across the resistance is

$$e = \text{impedance, due to resistance,} \times i$$
$$= (R + j\,\text{zero}) \times i$$
$$= (R + j\,\text{zero}) \times I \sin\left(pt - \tan^{-1}\frac{pL}{R}\right)$$
$$= \sqrt{R^2 + \text{zero}^2}\; I \sin\left(pt - \tan^{-1}\frac{pL}{R} + \tan^{-1}\frac{\text{zero}}{R}\right)$$
$$= R\,I \sin\left(pt - \tan^{-1}\frac{pL}{R} + \text{zero}\right)$$
$$= R\,I \sin\left(pt - \tan^{-1}\frac{pL}{R}\right) \qquad (14)$$

which gives the magnitude as well as the phase angle of the resistance component of the impressed voltage

(*c*) The inductance component of the impressed voltage will be obtained as follows —

The current is given by equation (13)

The impedance vector due to the inductance of the circuit $=$ zero $+ j\,pL$

Therefore, the voltage across the inductance will be

$$e_L = \text{impedance} \times i =$$
$$(\text{zero} + j\,pL) \times I \sin\left(pt - \tan^{-1}\frac{pL}{R}\right)$$

$$= \sqrt{\text{zero}^2 + p^2 L^2} \times$$

$$I \sin\left(pt - \tan^{-1}\frac{pL}{R} + \tan^{-1}\frac{pI}{\text{zero}}\right)$$

$$= pLI \sin\left(pt - \tan^{-1}\frac{pI}{R} + 90\right) \quad (15),$$

which gives both magnitude and phase angle of the inductance component of the impressed voltage

We can now summarise the above results as follows —

In a circuit containing resistance and inductance connected in series, the following relations will exist between the current and the various voltages —

(A) If the current flowing through the circuit is represented by

$$i = I \sin pt \qquad (a)$$

the impressed voltage will be

$$e = \sqrt{R^2 + p^2 L^2} \times$$

$$I \sin\left(pt + \tan^{-}\frac{pL}{R}\right) \qquad (b)$$

The resistance component of the impressed voltage will be

$$e = R I \sin pt \qquad (c)$$

The inductance component of the impressed voltage will be

$$e_L = pLI \sin(pt + 90) \qquad (d)$$

(B) If the impressed voltage is represented by

$$e = E \sin pt \qquad (a_1),$$

the current will be

$$i = I \sin\left(pt - \tan^{-1}\frac{pL}{R}\right) \qquad (b_1),$$

where $I = E \div \sqrt{R^2 + p^2 L^2}$

The resistance component of the impressed voltage will be

$$e_R = R\,I \sin\left(pt - \tan^{-1}\frac{pL}{R}\right) \qquad (c_1)$$

The inductance component of the impressed voltage will be

$$e_L = p\,L\,I \sin\left(pt - \tan^{-1}\frac{pL}{R} + 90^\circ\right) \qquad (d_1)$$

It will be noted, that in both cases the current is found to be lagging behind the voltage. The greater is p L than R, the greater will be the angle of lag. Also, the resistance component of the impressed voltage is in phase with the current, and the inductance component of the impressed voltage is 90° in advance of the current.

37. Current and Voltage Sine Waves in a Circuit containing Capacity only.—We will now consider the case of a circuit which possesses capacity only. Suppose, such a circuit is impressed with an alternating voltage represented by

$$e = E \sin (pt - 90) \qquad (1),$$

and let the capacity of the circuit = C fds

We know that the current flowing in this circuit will be

$$i = C \frac{de}{dt} = C \frac{d\, E \sin (pt - 90)}{dt}$$

$$= C\, p\, E \sin pt$$

$$= I \sin pt \qquad (2),$$

where $I = C\, p\, E$

We can see that the current is leading the voltage by 90, and that the maximum value of current, flowing in a circuit comprising a capacity of C fds and impressed with a voltage sine wave of maximum value E, is $I = p\, C\, E$ Therefore, the impedance of such a circuit, expressed symbolically, will be

$$\frac{1}{j\, p\, C} = \frac{j}{j\, p\, C \times j} = \frac{j}{j^2\, p\, C} = \frac{-j}{p\, C}$$

$$= -j \frac{1}{p\, C} \text{ or}$$

$$= \text{zero} + j \left(-\frac{1}{p\, C}\right) \qquad (3)$$

Having found the symbolic expression for the impedance of a circuit containing only capacity of C fds, we can easily find the magnitude and the phase angle of the impressed voltage if the current is given or *vice versa*

For instance, let the impressed voltage be

$$e = E \sin (pt - 90)$$

The impedance of the circuit containing only capacity $= C$ fds will be

$$\text{zero} + j\left(p\ \text{zero} - \frac{1}{pC}\right)$$

$$= \text{zero} + j\left(-\frac{1}{pC}\right),$$ since resistance and inductance are both zero

Therefore, the current in the circuit $=$

$$i = \frac{\text{impressed voltage}}{\text{impedance}} = \frac{e}{\text{impedance}}$$

$$= \frac{E \sin (pt - 90)}{\text{zero} + j\left(-\frac{1}{pC}\right)}$$

$$= \frac{E}{\sqrt{\text{zero}^2 + \left(-\frac{1}{pC}\right)^2}} \times$$

$$\sin\left\{pt - 90 - \tan^{-1}\left(\frac{-\frac{1}{pC}}{\text{zero}}\right)\right\}$$

$$= \frac{E}{\frac{1}{pC}} \sin (pt - 90 + \tan^{-1} \infty)$$

$$= pCE \sin (pt - 90 + 90)$$

$$= pCE \sin pt$$

$$= I \sin pt \qquad (4),$$

where $I = pCE$.

Therefore, the maximum value of the current is $I = p\,C\,E$, and it is 90° in advance of the voltage. The phase position, obtained for the current, establishes the fact that in a circuit which contains only capacity, the current leads the impressed voltage by 90°.

38. **Current and Voltage Sine Waves in a Circuit possessing Resistance Inductance and Capacity joined in Series.**—Let us next study the case of an electric circuit which possesses resistance $= R$ ohms, inductance $= L$ henries, and capacity $= C$ fds, all connected in series. Let the current flowing through the circuit be expressed by

$$i = I \sin pt \qquad (1)$$

The magnitude and phase angle of the impressed voltage and its components will be found as follows:—

(a) To find the magnitude and phase angle of the impressed voltage.

The current is given by equation (1).

The impedance vector of the circuit, expressed symbolically, will be represented by

$$R + j\left(p\,L - \frac{1}{p\,C}\right) \qquad (2)$$

Therefore, the impressed voltage at any instant =

$$e = \text{impedance} \times \text{current at that instant}$$

$$= \left[R + j\left(pL - \frac{1}{pC}\right) \right] \times i$$

$$= \left[R + j\left(pL - \frac{1}{pC}\right) \right] \times I \sin pt$$

Multiplying, according to the rules established in the preceding chapter, I sin pt by

$R + j\left(pL - \frac{1}{pC}\right)$, we get

$$e = \sqrt{R^2 + \left(pL - \frac{1}{pC}\right)} \times$$

$$I \sin\left(pt + \tan^{-1}\frac{pL - \frac{1}{pC}}{R}\right)$$

$$= E \sin\left(pt + \tan^{-1}\frac{pL - \frac{1}{pC}}{R}\right) \qquad (3),$$

$$\text{where } E = \sqrt{R^2 + \left(pL - \frac{1}{pC}\right)^2} \times I$$

The analytical equation (3) expresses, both in magnitude and phase angle, the impressed voltage The following important results are to be particularly noted

(*i*) The impressed voltage will be in phase with the current if $\left(pL - \frac{1}{pC}\right) = \text{zero}$ The expression for the voltage will then become

$e = \mathrm{R\ I} \sin pt$

This shows that in an alternating current circuit which contains resistance, inductance, and capacity connected in series, the circuit behaves as if it contained only resistance when the inductive reactance equals the capacity reactance, *i e* when $p\,\mathrm{L} = \frac{1}{p\,\mathrm{C}}$ The current therefore will be in phase with the voltage

(*ii*) The impressed voltage will be in advance of the current when $p\,\mathrm{L}$ is greater than $\frac{1}{p\,\mathrm{C}}$, *i e* when the effect of inductance is greater than the effect due to capacity

(*iii*) The impressed voltage will be lagging behind the current when $p\,\mathrm{L}$ is less than $\frac{1}{p\,\mathrm{C}}$, *i e* when the effect due to inductance is smaller than the capacity effect

(*b*) *To find the magnitude and phase angle of the resistance component of the impressed voltage*

The current is given by equation (1) The impedance vector of the resistance portion of the circuit, symbolically expressed, will be

$$R + j\left(p\,\text{zero} - \frac{1}{p\,\text{zero}}\right)$$

$$= R + j\,\text{zero} \qquad (4)$$

Therefore, the resistance component of the impressed voltage will be

e = impedance vector due to resistance × current

$$= (R + j\,\text{zero}) \times i$$

$$= (R + j\,\text{zero})\, I \sin pt$$

$$= \sqrt{R^2 + \text{zero}^2}\; I \sin\left(pt + \tan^{-1}\frac{\text{zero}}{R}\right)$$

$$= R\, I \sin pt$$

$$= E_r \sin pt \qquad (5),$$

where $E_r = R\, I$

The equation (5) gives the magnitude and phase angle of the voltage across the resistance It is in phase with the current

(*c*) *To find the magnitude and phase angle of the inductance component of the impressed voltage*

The current is given by equation (1) The impedance vector for the inductance, symbolically expressed, will be represented by

$$\text{zero} + j\left(p\,L - \frac{1}{p\ \text{zero}}\right)$$

infinity

$$= \text{zero} + j\,p\,L \qquad (6)$$

Therefore, the inductance component of the impressed voltage will be

$$e_L = \text{impedance due to inductance} \times \text{current}$$
$$= (\text{zero} + j\,p\,L) \times i$$
$$= (\text{zero} + j\,p\,L)\ I \sin pt$$
$$= \sqrt{\text{zero}^2 + p^2 L^2}\ I \sin\left(pt + \tan^{-1}\frac{p\,L}{\text{zero}}\right)$$
$$= p\,L\,I \sin(pt + 90^\circ)$$
$$= E_L \sin(pt + 90^\circ) \qquad (7),$$

where $E_L = p\,L\,I$

The equation (7) gives the magnitude and phase angle of the voltage across the inductance It is 90° in advance of the current

(*d*) *To find the magnitude and phase angle of the capacity component of the impressed voltage*

The current is given by equation (1) The impedance vector of the capacity portion of the circuit, expressed symbolically, will be

$$= \text{zero} + j\left(p\ \text{zero} - \frac{1}{p\,C}\right)$$

$$= \text{zero} + j\left(-\frac{1}{p\,C}\right) \qquad (8)$$

Therefore, the voltage across the capacity will be

e_c = impedance vector due to capacity × current

$$= \left(\text{zero} + j\,\frac{1}{-p\,C}\right) \times i$$

$$= \left(\text{zero} + j\,\frac{1}{-p\,C}\right) I \sin pt$$

$$= \sqrt{\text{zero}^2 + \left(\frac{1}{-p\,C}\right)^2} \times$$

$$I \sin\left(pt + \tan^{-1}\frac{-\frac{1}{p\,C}}{\text{zero}}\right)$$

$$= \frac{I}{p\,C} \sin(pt - 90^\circ)$$

$$= E \sin(pt - 90^\circ) \qquad (9),$$

where $E = \dfrac{I}{p\,C}$

The equation (9) gives the magnitude as well as the phase angle of the voltage across the capacity

We observe from the above results that if we take the current as our standard vector of reference, since it lies in the positive horizontal direction, and measure the phase angle of voltages with respect to it, we find, that

(*a*) the impressed voltage is either in phase with, in advance of, or lags behind the current, depending upon whether the inductive reactance is equal to, greater, or smaller than the capacity reactance

(*b*) the voltage across the resistance portion of the circuit is in phase with the current

(*c*) the voltage across the inductance portion of the circuit is 90° in advance of the current

(*d*) the voltage across the capacity portion of the circuit is 90 lagging behind the current

The student will note from the above discussion, that a clear understanding of the behaviour of alternating currents and voltages in circuits containing resistance, inductance and capacity is very essential for a thorough grasp of all classes of alternating current work He is, therefore, strongly advised to study very critically the results obtained above He should work out, as an exercise, maximum value, *r m s* value and phase angle of impressed voltage and its components for a given value of current and assuming various values and

arrangements of connections of resistance, inductance and capacity

39 **Series and Parallel Arrangements of Impedances are added like Resistances** In our application of the symbolic method, in the above illustrations, for the determination of magnitude and phase angles of currents and voltages, it will be noted that we had expressed the impedance vectors symbolically In the last section, we had illustrated the case of an electric circuit in which resistance, inductance and capacity were arranged in series, and in arriving at their total impedance we added the impedance vectors like resistances If the various impedances had been arranged in parallel, their impedance vectors should also have been compounded according to the rule applicable to the addition of resistances arranged in parallel

An easy way to prove the above is as follows —

(*a*) *Series arrangement of Impedances*

If we have a series arrangement of resistance, inductance and capacity, and if x_1, x_2 and x_3 represent the impedance vectors of resistance, inductance and capacity respectively, and if the current flowing through

the circuit is $i = I \sin pt$, the voltages across resistance, inductance and capacity will be e_1, e_2 and e_3 respectively, where

$$e_1 = x_1 \, I \sin pt,$$
$$e_2 = x_2 \, I \sin pt, \text{ and}$$
$$e_3 = x_3 \, I \sin pt$$

Therefore, the resultant voltage vector will be

$$e_1 + e_2 + e_3$$
$$= x_1 I \sin pt + x_2 I \sin pt + x_3 I \sin pt$$
$$= (x_1 + x_2 + x_3) \, I \sin pt$$

But $x_1 + x_2 + x_3$ is the sum of the separate impedance vectors. It is, therefore, clear that when the impedances are joined in series, they combine like resistances.

(*b*) *Parallel arrangement of Impedances*

Let resistance, inductance and capacity be connected in parallel, and let their impedance vectors be represented by x_1, x_2 and x_3 respectively. If i_1, i_2 and i_3 are the corresponding currents flowing through the branch circuits, and if the voltage across the circuit is

$$e = E \sin pt \qquad (1),$$

then,

$$i_1 = \frac{E \sin pt}{x_1}$$

$$i_2 = \frac{E \sin pt}{x_2}$$

$$i_3 = \frac{E \sin pt}{x_3}$$

Therefore, the resultant current

$$= i_1 + i_2 + i_3$$

$$= \frac{E}{x_1} \sin pt + \frac{E}{x_2} \sin pt + \frac{E}{x_3} \sin pt$$

$$= \left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}\right) E \sin pt$$

$$= \frac{1}{\dfrac{1}{\left(\dfrac{1}{x_1} + \dfrac{1}{x_2} + \dfrac{1}{x_3}\right)}} E \sin pt \quad (2)$$

It is evident from equation (2), that the resultant impedance vector of a circuit, containing resistance, inductance and capacity arranged in parallel, is the reciprocal of the sum of the reciprocals of the branch impedances The impedance vectors, even if connected in parallel, are therefore combined like resistances

40 **Symbolic Expressions for Impedance Vectors** —We have observed, in the foregoing illustrations of the application of the symbolic method for the determination of the magnitudes and phase angles of voltages and currents entering

$$i_1 = \frac{E \sin pt}{x_1}$$

$$i_2 = \frac{E \sin pt}{x_2}$$

$$i_3 = \frac{E \sin pt}{x_3}$$

Therefore, the resultant current

$$= i_1 + i_2 + i_3$$

$$= \frac{E}{x_1} \sin pt + \frac{E}{x_2} \sin pt + \frac{E}{x_3} \sin pt$$

$$= \left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}\right) E \sin pt$$

$$= \frac{1}{\dfrac{1}{\left(\dfrac{1}{x_1} + \dfrac{1}{x_2} + \dfrac{1}{x_3}\right)}} E \sin pt \qquad (2)$$

It is evident from equation (2), that the resultant impedance vector of a circuit, containing resistance, inductance and capacity arranged in parallel, is the reciprocal of the sum of the reciprocals of the branch impedances The impedance vectors, even if connected in parallel, are therefore combined like resistances

40 **Symbolic Expressions for Impedance Vectors**—We have observed, in the foregoing illustrations of the application of the symbolic method for the determination of the magnitudes and phase angles of voltages and currents entering

into a problem, that it is essential to express the impedance vectors symbolically for converting voltage vector into current vector, or *vice versa* We may, therefore, summarise the symbolic expressions for the impedance vectors of some of the most important combinations of resistance, inductance and capacity as follows —

(1) The circuit contains only resistance = R ohms,

the impedance vector of the circuit

$$= R + j \text{ zero} \qquad (1)$$

(2) The circuit contains only inductance = L henries,

the impedance vector of the circuit

$$= \text{zero} + j\, p L \qquad (2)$$

(3) The circuit contains only capacity = C fds,

the impedance vector of the circuit

$$= \text{zero} + j\left(-\frac{1}{pC}\right) \qquad (3)$$

(4) The circuit contains resistance = R ohms and inductance = L henries, both joined in series,

the impedance vector of the circuit

$$= R + j\, p L \qquad (4)$$

(5) The circuit contains resistance = R ohms and capacity = C fds, both joined in series,

the impedance vector of the circuit

$$= R + j\left(-\frac{1}{pC}\right) \qquad (5)$$

(6) The circuit contains inductance = L henries and capacity = C fds, both joined in series,

the impedance vector of the circuit

$$= \text{zero} + j\left(pL - \frac{1}{pC}\right) \qquad (6)$$

(7) The circuit contains resistance = R ohms, inductance = L henries and capacity = C fds, joined in series,

the impedance vector of the circuit

$$= R + j\left(pL - \frac{1}{pC}\right) \qquad (7)$$

(8) The circuit contains resistance = R ohms and inductance = L henries joined in parallel

the resultant impedance vector of the circuit will be

$$= 1 \div \left(\frac{1}{R} + \frac{1}{jpL}\right)$$

$$= 1 \div \left(\frac{1}{R} + j\,\frac{1}{-pL}\right),$$

$$= \frac{1}{\frac{1}{R} + j\left(\frac{1}{-pL}\right)} \qquad (8)$$

(9) The circuit contains resistance = R ohms and capacity = C fds, joined in parallel, the resultant impedance vector of the circuit

$$= 1 - \left(\frac{1}{R} + \frac{1}{\frac{j}{-pC}}\right)$$

$$= 1 - \left(\frac{1}{R} - \frac{pC}{j}\right)$$

$$= 1 - \left(\frac{1}{R} + jpC\right)$$

$$= \frac{1}{\frac{1}{R} + jpC} \qquad (9)$$

(10) The circuit contains inductance = L henries and capacity = C fds, joined in parallel,

the resultant impedance vector of the circuit

$$= 1 - \left(\frac{1}{jpL} + \frac{1}{\frac{j}{-pC}}\right)$$

$$= 1 - \left(\frac{1}{jpL} + jpC\right)$$

$$= 1 - \left(\frac{-j}{p\,L} + j\,p\,C\right)$$

$$= 1 - \left[j\left(p\,C - \frac{1}{p\,L}\right) \right]$$

$$= \frac{1}{\text{zero} + j\left(p\,C - \frac{1}{p\,L}\right)} \qquad (10)$$

(11) The circuit contains resistance $= R$ ohms, inductance $= L$ henries and capacity $= C$ fds, joined in parallel, the resultant impedance vector of the circuit

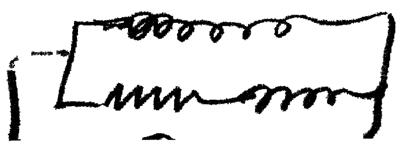
$$= 1 - \left(\frac{1}{R} + \frac{1}{j\,p\,L} + \frac{1}{\frac{j}{-p\,C}}\right)$$

$$= 1 - \left(\frac{1}{R} - j\,\frac{1}{p\,L} + j\,p\,C\right)$$

$$= 1 - \left[\frac{1}{R} + j\left(p\,C - \frac{1}{p\,L}\right)\right]$$

$$= \frac{1}{\frac{1}{R} + j\left(p\,C - \frac{1}{p\,L}\right)} \qquad (11)$$

Example Let us take an example and apply the results of the preceding discussion Suppose, we have an electric circuit which comprises two branch circuits *a* and *b* respectively, and they are joined in parallel Let the



branch circuit *a* po sess only inductance of L_1 henries, and the branch circuit *b* contains re sistance of R ohms, and inductance of L_2 henries joined in series Let the current supplied by the alternator to the circuit be expressible by

$i = I \sin pt$ (1),

and we are required to determine

(i) the magnitude and phase angle of the impressed voltage and

(ii) the magnitudes and phase angles of currents in branch circuits

We note from the above data that we are given the main current and it is expressible by $i = I \sin pt$ As the vector lies in the positive horizontal direction, we can take it as a vector of reference for the measurement of the phase angles of the voltage vector and the vectors representing the branch currents

The impedance vector of the branch circuit *a* , which contains only inductance of L_1 hen ries is expressible by

zero $+ j p L_1$ (2)

The impendance vector of the branch circuit *b* , which contains resistance = R ohms, and in ductance = L_2 henries, is expressible by

$R + j pL_2$ (3)

Since the two impedances are in parallel,

the resultant impedance vector of the whole circuit will be the reciprocal of the sum of the reciprocals of each, *i e* they will be compounded like resistances

Therefore, the resultant impedance vector

$$= 1 - \left[\frac{1}{\text{zero} + j\,p\,L_1} + \frac{1}{R + j\,p\,L_2} \right]$$

$$= 1 - \left[\frac{R + j\,p\,L_2 + j\,p\,L_1}{(\text{zero} + j\,p\,L_1) \times (R + j\,p\,L_2)} \right]$$

$$= 1 - \frac{R + j\,(p\,L_1 + p\,L_2)}{(\text{zero} + j\,p\,L_1)\,(R + j\,p\,L_2)}$$

$$= \frac{(\text{zero} + j\,p\,I_1)\,(R + j\,p\,L_2)}{R + j\,(p\,L_1 + p\,L_2)} \qquad (4)$$

Having found the impedance vector for the branch circuits and also the resultant impedance vector, we can determine the impressed voltage and branch currents as follows —

(*a*) *Impressed voltage* —

The impressed voltage = resultant impedance vector × Main Current

$$= \frac{(\text{zero} + j\,p\,L_1)\,(R + j\,p\,L\ )}{R + j\,(p\,I_1 + p\,L_2)} \times 1$$

$$= \frac{(\text{zero} + j\,p\,L_1)\,(R + j\,p\,L\ )}{R + j\,(p\,L_1 + p\,L_2)}\,I \sin pt$$

$$= I \times \frac{\sqrt{\text{zero}^2 + p^2 L_1^2} \times \sqrt{R + p\ L_2}}{\sqrt{R^2 + (p\ L_1 + p\ L_2)^2}}$$

$$\times \sin\left(pt + \tan^{-1}\frac{p\ L_1}{\text{zero}} \right.$$

$$+ \tan^{-1}\frac{p\ I}{R}$$

$$\left. - \tan^{-1}\frac{(p\ L_1 + p\ L_2)}{R} \right)$$

$$= I \times \frac{\sqrt{\text{zero}^2 + p^2 L_1^2} \times \sqrt{R^2 + p^2 I_2}}{\sqrt{R^2 + (p\ L_1 + p\ L_2)^2}}$$

$$\times \sin\left(pt + 90 + \tan^{-1}\frac{p\ I}{R} \right.$$

$$\left. - \tan^{-1}\frac{p\ L_1 + p\ L_2}{R} \right)$$

or $e = E \sin (pt + \theta)$ (5),

$$\text{where } E = I \times \frac{\sqrt{\text{zero}^2 + p^2 L_1^2} \times \sqrt{R^2 + p^2 L_2}}{\sqrt{R^2 + (p\ L_1 + p\ L_2)^2}},$$

$$\text{and } \theta = \left(90 + \tan^{-1}\frac{p\ L}{R} - \tan^{-1}\frac{pL_1 + pI}{R}\right),$$

which gives both magnitude and phase angle of the impressed voltage

(*b*) *Current in branch circuit a*

$$= i = \frac{\text{impressed voltage}}{\text{impedance vector of branch circuit } a}$$

$$= \frac{E \sin (pt + \theta)}{\text{zero} + j\ p\ L_1}$$

$$=\frac{E}{\sqrt{\text{zero}^2 + p\ L_1}} \times$$

$$\sin\left(pt + \theta - \tan^{-1}\frac{p\,L_1}{\text{zero}}\right)$$

$$= \frac{E}{p\,L_1} \sin\left(pt + \theta - 90\ \right)$$

$$= I\ \sin\left(pt + \theta - 90\ \right) \qquad (6),$$

where $I = \frac{E}{p\,L_1}$

(*c*) *Current in branch circu t b*

$$= I_b = \frac{\text{impressed voltage}}{\text{impedance vector of branch circuit } b}$$

$$= \frac{E \sin\left(pt + \theta\ \right)}{R + j\,p\,L_2}$$

$$= \frac{E}{\sqrt{R\ + p^2\,L_2^2}} \times$$

$$\sin\left(pt + \theta - \tan^{-1}\frac{p\,L\ }{R}\right)$$

$$= I_b \sin\left(pt + \theta - \tan^{-1}\frac{p\,L\ }{R}\right) \qquad (7)$$

where $I_b = \frac{E}{\sqrt{R^2 + p^2\,L_2^2}}$

It will be observed from equations 5, 6 and 7, that

(*i*) The impressed voltage is leading the main current by an angle θ

(*ii*) The impressed voltage is leading the branch current a by 90°

(*iii*) The impressed voltage is also leading the branch current b by an angle less than 90° This is due to the fact that branch circuit b possesses resistance also which tends to reduce the angle of lag

The student is advised to take several examples of circuits containing resistance, inductance and capacity arranged in various ways, and find the magnitude and the phase angle of currents and voltages, entering into the problem, for a given magnitude and phase angle of either the impressed voltage or the current He should notice, in these exercises, several interesting points which would be very helpful to him in interpreting the electrical resonance phenomena so well known in alternating current circuits

CHAPTER VII

NUMERICAL EXAMPLES

In the preceding chapter, we had illustrated the application of the symbolic method to alternating current circuits, but there we did not give any numerical values to currents, voltages and impedances In this chapter, we would give numerical values and solve, in full detail, several examples, beginning with simple cases and gradually progressing towards complicated ones This would serve to illustrate the application of the symbolic method discussed in last chapter

Fundamental Rules to be observed — In all examples where the results are to be shown on a vector diagram, the student should make it a standard practice to observe the following rules —

(1) That the vector of reference, with respect to which all other vectors are measured for their phase angles, lies in the positive horizontal direction

(ii) That the vector, representing an alternating current or voltage sine wave, rotates in counter clockwise direction Therefore, a vector is turned counter clockwise for positive angles and clockwise for negative angles

(iii) For the purpose of representation on a vector diagram, an alternating current or voltage sine wave vector is imagined to be stationary and is therefore drawn, on a vector diagram, for a value of time t = zero

(iv) It therefore follows, that an alternating current sine wave of the form $i = I \sin pt$, or an alternating voltage sine wave of the form $e = E \sin pt$, will lie in the positive horizontal direction when plotted on a vector diagram

(v) An alternating current sine wave of the form $i = I \sin (pt + \theta)$, or an alternating voltage sine wave of the form $e = E \sin (pt + \theta)$, will both be drawn, when represented vectorially, $+\theta$ in advance of the vectors $i = I \sin pt$ and $e = E \sin pt$ respectively, *i e*, in counter clock

wise direction Their phase angles, therefore, will be measured $+\theta$ in advance of the positive horizontal direction

(ii) Similarly, an alternating current sine wave of the form $i = I \sin(pt - \theta)$, or an alternating voltage sine wave of the form $e = E \sin(pt - \theta)$ will each be drawn in clockwise direction at an angle θ from the positive horizontal direction

Examples — (Note—In the following examples we have used the slide rule as a guide for multiplications and division and therefore the values may not be absolutely accurate It is left to the student to work out again the following examples and determine the exact value)

EXAMPLE 1 —If an alternating voltage sine wave, at the consumer s terminals is expressible by $e = 339 \sin pt$ find the maximum value and also the r m s value of the voltage Which of the two values is referred to in commercial practice ?

Ans —*(a)* Maximum value of voltage

$= 339$ volts

and r m s value of voltage
$= 339 \times 707 = 240$ volts

(*b*) It is the r m s value and not the maximum value which is generally referred to in commercial practice to denote the strength of current or voltage

EXAMPLE 2 —Draw the following on a vector diagram —

(*a*) $e = 239 \sin pt$

(*b*) $e = 239 \sin (pt + 45^{\circ})$

(*c*) $e = 239 \sin (pt - 45)$

(*d*) $i = 50 \sin pt$

(*e*) $i = 50 \sin (pt + 30)$

(*f*) $i = 50 \sin (pt - 30)$

Ans —The above vectors have been drawn and are shown in Fig 17

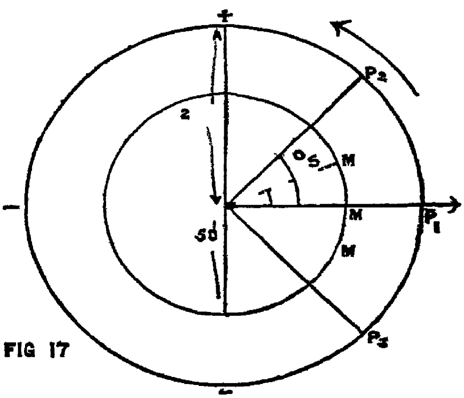


FIG 17

EXAMPLE 3—State the phase angles, when measured with respect to voltage (*a*), of voltages (*b*) and (*c*), and currents (*d*), (*e*) and (*f*) of the above example

Ans—The following are the phase angles of voltages and currents with respect to voltage e = 239 sin *pt* —

The voltage (*b*) is 45 in advance of the voltage e = 239 sin *pt*

The voltage (*c*) is 45 lagging behind the voltage e = 239 sin *pt*

The current (*d*) is in phase with the voltage e = 239 sin *pt*

The current (*e*) is 30 in advance of the voltage e = 239 sin *pt*

The current (*f*) is 30 lagging behind the voltage e = 239 sin *pt*

EXAMPLE 4—The voltage impressed on a circuit, having only a resistance of 4 ohms, follows the law e = 282 sin *pt*, where frequency f = 50 Find the maximum value, the r m s value and the phase angle of the current flowing through the circuit

Ans —The voltage is expressible by

$$e = 282 \sin pt \qquad (1)$$

and the impedance vector $= R + j$ zero

$$= 4 + j \text{ zero} \qquad (2)$$

Therefore, the current =

$$i = \frac{\text{voltage}}{\text{impedance}} = \frac{e}{4 + j \text{ zero}}$$

$$= \frac{282 \sin pt}{4 + j \text{ zero}}$$

$$= \frac{282}{\sqrt{4^2 + \text{zero}}} \sin\left(pt - \tan^{-1}\frac{\text{zero}}{4}\right)$$

$$= \frac{282}{4} \sin(pt - \text{zero})$$

$$= 70\ 5 \sin pt \qquad (3)$$

Therefore, the maximum value of current =
70 5 amperes,

and the r m s value of current = 70 5 × 707
= 49 84 amperes

The current is in phase with the voltage

We may again draw the attention of the student as to how the operation of converting the voltage vector into a current vector has been carried out It should be noted that the voltage vector was expressed analytically, and

the impedance vector alone had been denoted symbolically The operation of dividing the voltage vector by the impedance vector R + j zero really means, dividing the magnitude of the voltage vector by $\sqrt{R^2 + \text{zero}}$ to get the magnitude of the current vector, and decreasing the phase angle of the voltage vector by an angle $\tan^{-1} \frac{\text{zero}}{R}$ to give the phase position of the new vector The advantage of keeping the voltage vector in its analytical form being that when it is operated by the impedance vector, the resulting current vector is obtained in the analytical form

EXAMPLE 5 —The current flowing through a circuit having only resistance of 4 ohms, follows the law $i = 70.5 \sin pt$, where frequency $f = 50$ Find the maximum value, the r m s value and the phase angle of the voltage driving the above current

Ans —The current is given by

$$i = 70.5 \sin pt \qquad (1)$$

The impedance vector = R + j zero

$$= 4 + j \text{ zero} \qquad (2)$$

Therefore, the impressed voltage

$$
\begin{aligned}
e &= \text{impedance vector} \times \text{current} \\
&= (4 + j\ \text{zero}) \times i \\
&= (4 + j\ \text{zero}) \times 70.5 \sin pt \\
&= \sqrt{4^2 + \text{zero}^2} \times 70.5 \sin \left(pt + \tan^{-1}\frac{\text{zero}}{4}\right) \\
&= 4 \times 70.5 \sin (pt + \text{zero}) \\
&= 282 \sin pt \qquad (3)
\end{aligned}
$$

Therefore, the maximum value of the voltage

$$= 282 \text{ Volts},$$

and the r. m. s. value of the voltage

$$
\begin{aligned}
&= 282 \times .707 \\
&= 200 \text{ Volts}.
\end{aligned}
$$

The voltage is in phase with the current.

The student should note that Example 4 and the above example are the same. In the former, the voltage was given and the current was to be determined, in the latter the current was known and the voltage was to be found.

EXAMPLE 6.—The circuit contains inductance only = 0.5 henries and the impressed voltage is expressible by $e = 282 \sin pt$, where frequency $f = 50$. Find

(*a*) maximum value of current,
(*b*) r m s value of current,
and (*c*) phase angle of current

Ans — The voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance of the circuit will be

$= R + j\,p\,L$, where $p = 2\pi f = 2\pi\ 50 =$
$3\ 14 \times 100 = 314$,

$$= \text{zero} + j\ 314 \times\ 05$$

$$= \text{zero} + j\ 15\ 7 \qquad (2)$$

Therefore, the current, flowing in the circuit,

$$i = \frac{\text{voltage}}{\text{impedance}} = \frac{e}{\text{zero} + j\ 15\ 7}$$

$$= \frac{282 \sin pt}{\text{zero} + j\ 15\ 7}$$

$$= \frac{282}{\sqrt{\text{zero}^2 + 15\ 7}} \sin\left(pt - \tan^{-1}\frac{15\ 7}{\text{zero}}\right)$$

$$= \frac{282}{15\ 7} \sin\left(pt - \tan^{-1} \infty\right)$$

$$= 18 \sin\left(pt - 90\ \right) \qquad (3)$$

Therefore,

(*a*) The maximum value of current $= 18$ amperes

(*b*) The r m s value of current $=$ $18 \times\ 707 = 13$ amperes

(*c*) The current lags 90 behind the voltage

EXAMPLE 7.—The current flowing through a circuit possessing only inductance of 0.5 henries is expressible by $i = 18 \sin (pt - 90^\circ)$, where frequency $f = 50$. Find

(*a*) the maximum value,

(*b*) the r. m. s. value and

(*c*) the phase angle of the impressed voltage.

Ans.—The current is given by

$$i = 18 \sin (pt - 90^\circ) \qquad (1)$$

The impedance of the circuit

$$= R + j\, pL$$

$$= \text{zero} + j\, 314 \times 0.5$$

$$= \text{zero} + j\, 157 \qquad (2)$$

Therefore, the impressed voltage =

$$e = \text{impedance} \times \text{current}$$

$$= (\text{zero} + j\, 157) \times i$$

$$= (\text{zero} + j\, 157) \times 18 \sin (pt - 90^\circ)$$

$$= \sqrt{\text{zero}^2 + 157^2}\ 18 \sin \left(pt - 90^\circ + \tan^{-1} \frac{157}{\text{zero}}\right)$$

$$= 157 \times 18 \sin (pt - 90^\circ + \tan^{-1} \infty)$$

$$= 282 \sin (pt - 90^\circ + 90^\circ)$$

$$= 282 \sin pt \qquad (3)$$

Therefore,

(*a*) the maximum value of the impressed voltage = 282 volts.

(b) the r m s value $= 282 \times 707$
$= 200$ volts

(c) the voltage is 90 in advance of the current in other words, the current is lagging behind the impressed voltage by 90

EXAMPLE 8 —A circuit contains capacity $C = 60 \times 10^{-6}$ farads The impressed voltage is expressible by $e = 282 \sin pt$, where frequency $f = 50$ Find

(a) the maximum value,

(b) the r m s value and

(c) the phase angle of the current flowing through the circuit

Ans —The impressed voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance of the circuit

$$= R + j\left(pL - \frac{1}{pC}\right)$$

$$= \text{zero} + j\left(\text{zero} - \frac{1}{pC}\right)$$

$$= \text{zero} + j\left(-\frac{1}{314 \times 60 \times 10^{-6}}\right)$$

$$= \text{zero} + j\left(-\frac{10^6}{18840}\right)$$

$$= \text{zero} + j(-53) \qquad (2)$$

Therefore, the current flowing through the circuit

$$= i = \frac{\text{voltage}}{\text{impedance}} = \frac{e}{\text{impedance}}$$

$$= \frac{282 \sin pt}{\text{zero} + j\,(-53\,)}$$

$$= \frac{282}{\sqrt{\text{zero}^2 + (-53\,)^2}} \times$$

$$\sin\left(pt - \tan^{-1}\frac{-53}{\text{zero}}\right)$$

$$= \frac{282}{53} \sin\left(pt + \tan^{-1}\frac{53}{\text{zero}}\right)$$

$$= 5\;32 \sin(pt + 90\;) \qquad (3)$$

Therefore,

(*a*) the maximum value of the current
= 5 32 amperes

(*b*) the r m s value of the current
= 5 32 × 707 = 3 76 amperes

(*c*) the current is 90 in advance of the voltage

Example 9 — A circuit contains capacity $C = 60 \times 10^{-6}$ farads The current flowing through the circuit is expressible by $i = 5\;32 \sin(pt + 90\;)$, where frequency f = 50 Find

(*a*) the maximum value,

(*b*) the r m s value, and

(*c*) the phase angle of the impressed voltage

Ans — The current is given by

$$i = 5.32 \sin (pt + 90^\circ) \qquad (1)$$

The impedance of the circuit

$$= R + j\left(pL - \frac{1}{pC}\right)$$

$$= \text{zero} + j\left(\text{zero} - \frac{1}{314 \times 60 \times 10^{-6}}\right)$$

$$= \text{zero} + j\left(-\frac{10^6}{18840}\right)$$

$$= \text{zero} + j(-53) \qquad (2)$$

Therefore, the impressed voltage across the circuit

$$= e = \text{impedance vector} \times \text{current}$$

$$= [\text{zero} + j(-53)] \times i$$

$$= [\text{zero} + j(-53)]\ 5.32 \sin (pt + 90^\circ)$$

$$= \sqrt{\text{zero}^2 + (-53)^2} \times$$

$$5.32 \sin \left(pt + 90^\circ + \tan^{-1} \frac{-53}{\text{zero}}\right)$$

$$= 53 \times 5.32 \sin \left(pt + 90^\circ - \tan^{-1} \frac{53}{\text{zero}}\right)$$

$$= 53 \times 5.32 \sin (pt + 90^\circ - 90^\circ)$$

$$= 53 \times 5.32 \sin pt$$

$$= 282 \sin pt \qquad (3)$$

Therefore,

(*a*) the maximum value of the impressed voltage $= 282$ volts

(*b*) the r m s value of the impressed voltage = 282 × 707 = 200 volts

(*c*) the voltage is lagging 90 behind the current, in other words, the current is leading the voltage by 90

EXAMPLE 10 —A circuit contains resistance = 4 ohms and inductance = 0ɔ henries, joined in series The circuit is connected across an alternating current mains, and the supply voltage is expressible by e = 282 sin *pt*, the frequency being ɔ0 Find

(*a*) the r m s value and the phase angle of the current flowing in the circuit,

(*b*) the r m s value and phase angle of the resistance component of the impressed voltage and

(*c*) the r m s value and phase angle of the inductance component of the impressed voltage

Ans —The impressed voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance vector of the whole circuit

$$= R + j\,p\,L = 4 + j\,ɔ14 \times 0ɔ$$

$$= 4 + j\,1ɔ\,7 \qquad (2)$$

The impedance vector of resistance

$= R + j\,p\,L = 4 + j\,p$ zero

$= 4 + j$ zero (3)

The impedance vector of inductance

$= R + j\,pL =$ zero $+ j\,p \times 05$

$=$ zero $+ j\ 314 \times 05$

$=$ zero $+ j\ 15\ 7$ (4)

(*a*) The current in the circuit

$$= i = \frac{e}{\text{impedance}} = \frac{282 \sin pt}{4 + j\ 15\ 7}$$

$$= \frac{282}{\sqrt{4 + 15\ 7^2}} \times$$

$$\sin\left(pt - \tan^{-1}\frac{15\ 7}{4}\right)$$

$$= \frac{282}{16\ 2} \sin\left(pt - \tan^{-1} 3\ 925\right)$$

$$= 17\ 4 \sin\left(pt - 75\ 42\ \right) \qquad (5)$$

Therefore, the r m s value of the current

$= 17\ 4 \times 707 = 12\ 3$ amperes,

and the current is lagging behind the voltage by 75 42 The student should note that the presence of resistance in series with inductance has reduced the angle of lag to 75 42 Had there been no resistance, the angle of lag would be 90

(*b*) The resistance component of the impressed voltage

$= e = \text{impedance due to resistance} \times \text{current}$

$= (4 + j \text{ zero}) \times i$

$= (4 + j \text{ zero}) \times 17\ 4 \sin(pt - 75\ 42\ )$

$= \sqrt{4^2 + \text{zero}^2} \times 17\ 4 \sin\left(pt - 75\ 42 + \tan^{-1}\frac{\text{zero}}{4}\right)$

$= 4 \times 17\ 4 \sin(pt - 75\ 42 + \text{zero})$

$= 69\ 6 \sin(pt - 75\ 42\ )$ (6)

Therefore, the r m s value of the resistance component of the impressed voltage

$= 69\ 6 \times\ 707$

$= 49\ 2$ volts,

and this voltage is in phase with the current, but it is lagging behind the impressed voltage by 75 42

(*c*) The inductance component of the impressed voltage =

$e_L = \text{impedance due to inductance} \times \text{current}$

$= (\text{zero} + j\ 15\ 7) \times i$

$= (\text{zero} + j\ 15\ 7) \times 17\ 4 \sin(pt - 75\ 42\ )$

$= \sqrt{\text{zero}^2 + 15\ 7} \times 17\ 4 \sin\left(pt - 75\ 42 + \tan^{-1}\frac{15\ 7}{\text{zero}}\right)$

$= 15\ 7 \times 17\ 4 \sin(pt - 75\ 42 + 90\ )$

$= 273 \sin(pt - 75\ 42 + 90\ )$ (7)

Therefore, the r m s value of the inductance component of the impressed voltage = 273 × 707 = 193 volts, and this voltage is 90 in advance of the current, 14 18 in advance of the impressed voltage and 90 in advance of the resistance component of the impressed voltage

The student should, as an exercise, assume that the current flowing through the circuit of the above example is given and expressible by $i = 17\ 4 \sin (pt - 75\ 42)$, and find out the r m s value and phase angle of the impressed voltage

EXAMPLE 11 — A circuit contains resistance = 4 ohms and capacity = 60×10^{-6} farads, joined in series The circuit is connected across an alternating voltage supply of 50 frequency The voltage is expressible by $e = 282 \sin pt$ Find the r m s value and phase angle of the current flowing in the circuit, and also the r m s value and phase angle of the components of the impressed voltage

Ans — The impressed voltage is given by

$$e = 282 \sin pt \tag{1}$$

The impedance of the circuit

$$= R + j\left(pL - \frac{1}{pC}\right)$$

$$= 4 + j\left(p \times \text{zero} - \frac{1}{p \times 60 \times 10^{-6}}\right)$$

$$= 4 + j\left(-\frac{1}{314 \times 60 \times 10^{-6}}\right)$$

$$= 4 + j\,(-53) \tag{2}$$

The impedance of the resistance portion of the circuit

$$= R + j\ \text{zero}$$

$$= 4 + j\ \text{zero} \tag{3}$$

The impedance of the capacity portion of the circuit

$$= \text{zero} + j(-53) \tag{4}$$

Therefore,

(*a*) The current in the circuit will be given by

$$i = \frac{\text{impressed voltage}}{\text{impedance of the circuit}}$$

$$= \frac{282 \sin pt}{4 + j\,(-53)}$$

$$= \frac{282}{\sqrt{4 + (-53)}} \times \sin\left(pt - \tan^{-1}\frac{-53}{4}\right)$$

$$= \frac{282}{53} \sin\left(pt + \tan^{-1} 13\ 25\right)$$

$$= 5\ 32 \sin\left(pt + 85\ 41\ \right) \qquad (5)$$

Therefore, the r m s value of current is $5\ 32 \times 707 = 3\ 76$ amperes, and the current is $85\ 41$ in advance of the impressed voltage

(*b*) The resistance component of the impressed voltage will be

$$= e_r = \text{impedance due to resistance} \times \text{current}$$

$$= (4 + j\ \text{zero}) \times 5\ 32 \sin\left(pt + 85\ 41\ \right)$$

$$= \sqrt{4^2 + \text{zero}^2} \times 5\ 32 \sin\left(pt + 85\ 41\ + \tan^{-1} \frac{\text{zero}}{4}\right)$$

$$= 4 \times 5\ 32 \sin\left(pt + 85\ 41\ + \text{zero}\right)$$

$$= 21\ 28 \sin\left(pt + 85\ 41\ \right) \qquad (6)$$

Therefore, the r m s value of the resistance component $= 21\ 28 \times 707 = 15\ 04$ volts It is in phase with the current, and $85^\circ\ 41$ in advance of the impressed voltage

(*c*) The capacity component of the impressed voltage will be

$$= e_c = \text{impedance due to capacity} \times \text{current}$$

$$= [\text{zero} + j\,(-\ 53\ )\ ] \times i$$

$$= [\text{zero} + j\,(-\ 53\ )\ ] \times 5\ 32 \sin\left(pt + 85\ 41\ \right)$$

$$= \sqrt{\text{zero}^2 + (-53)^2} \times 5\ 32$$

$$\sin\left(pt + 85\ 41 + \tan^{-1}\frac{-53}{\text{zero}}\right)$$

$$= 53 \times 5\ 32 \sin(pt + 85\ 41 - 90)$$

$$= 282 \sin(pt + 85\ 41 - 90) \qquad (7)$$

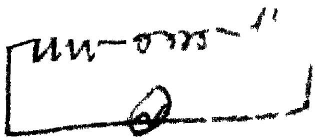
Therefore, the r m s value of the capacity component of the impressed voltage
$= 282 \times 707 = 200$ volts

It is 90 lagging behind the current, and 4 19 lagging behind the impressed voltage

The student should draw, on a vector diagram, current and voltages of the above problem, and study and think out for himself as to why they differ in phase angles It may be pointed out here that the impressed voltage and the capacity component of the impressed voltage are shown above to be equal in magnitude, while the resistance component of the impressed voltage is 21 28 volts maximum value This is due to the fact that the resistance component of the impressed voltage is very small compared with the capacity component, and the two being 90 apart in phase position, their resultant, *i e* the impressed voltage equals, for all practical purposes, the capacity component The student should work out the same exercise giving a higher value to the resistance, and he would then get

a greater difference in the magnitudes of the impressed voltage and its capacity component

EXAMPLE 12.—A circuit contains resistance = 4 ohms, inductance = 0·05 henries, and capacity = 60 $\times 10^{-6}$ farads, joined in series The circuit is connected across an alternating voltage supply which is expressible by e = 282 sin pt, where frequency f = 50 Find the r m s value and phase angle of the current flowing in the circuit and also the r m s value and phase angle of the components of the impressed voltage



Ans—The impressed voltage is given by

$$e = 282 \sin pt \tag{1}$$

The impedance of the whole circuit

$$= R + j\left(pL - \frac{1}{pC}\right)$$

$$= 4 + j\left(314 \times 0{\cdot}05 - \frac{1}{314 \times 60 \times 10^{-6}}\right)$$

$$= 4 + j\,(15{\cdot}7 - 53)$$

$$= 4 + j\,(-37{\cdot}3) \tag{2}$$

The impedance due to resistance

$$= R + j \text{ zero}$$

$$= 4 + j \text{ zero} \tag{3}$$

The impedance due to inductance

$$= \text{zero} + j\, p\, L$$

$$= \text{zero} + j\ 15.7 \tag{4}$$

The impedance due to capacity

$$= \text{zero} + j\left(\text{zero} - \frac{1}{p\,C}\right)$$

$$= \text{zero} + j\,(-53) \tag{5}$$

Therefore,

(*a*) The current flowing in the circuit

$$= i = \frac{\text{impressed voltage}}{\text{impedance of the circuit}}$$

$$= \frac{e}{4 + j\,(-37.3)} = \frac{282 \sin pt}{4 + j\,(-37.3)}$$

$$= \frac{282}{\sqrt{4^2 + (-37.3)^2}} \times$$

$$\sin\left(pt - \tan^{-1}\frac{-37.3}{4}\right)$$

$$= \frac{282}{37.3} \sin\,(pt + \tan^{-1} 9.3)$$

$$= 7.56 \sin\,(pt + 83^\circ\ 10') \tag{6}$$

Therefore, the r m s value of the current

$= 7\ 56 \times\ 707 = 5\ 35$ amperes, and the current is $83^\circ\ 10$ in advance of the impressed voltage

(*b*) The resistance component of the impressed voltage

$= e =$ impedance due to resistance $\times$ current

$= (4 + j \text{ zero}) \times 7\ 56 \times \sin(pt + 83^\circ\ 10)$

$= \sqrt{4^2 + \text{zero}^2} \times 7\ 56$

$\sin\left(pt + 83\ 10 + \tan^{-1}\frac{\text{zero}}{4}\right)$

$= 4 \times 7\ 56 \sin(pt + 83\ 10 + \text{zero})$

$= 30\ 24 \sin(pt + 83\ 10) \qquad (7)$

Therefore, the r m s value of the resistance component of the impressed voltage $= 30\ 24 \times\ 707 = 21\ 38$ volts It is in phase with the current, and 83 10 in advance of the impressed voltage

(*c*) The inductance component of the impressed voltage $=$

$e_L =$ impedance due to inductance $\times$ current

$= (\text{zero} + j\ 15\ 7) \times 7\ 56$

$\sin(pt + 83\ 10)$

$= \sqrt{\text{zero}^2 + (15\ 7)^2} \times 7\ 56 \sin\Big(pt$

$+ 83\ 10 + \tan^{-1}\frac{15\ 7}{\text{zero}}\Big)$

$= 15\,7 \times 7\,56 \sin(pt + 83\ 10 + 90\ )$

$= 118\,7 \sin(pt + 83\ 10 + 90\ ) \quad (8)$

Therefore, the r m s value of the inductance component of the impressed voltage = 118 7 × 707 = 83 92 volts It is 90 in advance of the current

(*d*) The capacity component of the impressed voltage =

e_c = impedance due to capacity × current

$= [\text{zero} + j(-53)] \times 7\,56 \sin(pt + 83\ 10\ )$

$= \sqrt{\text{zero}^2 + (-53)^2} \times 7\,56 \sin\left(pt - 83\ 10 + \tan^{-1}\dfrac{-53}{\text{zero}}\right)$

$= 53 \times 7\,56 \sin(pt + 83\ 10 - \tan^{-1}\infty)$

$= 400\,7 \sin(pt + 83\ 10 - 90\ )$

$= 400\,7 \sin(pt + 83\ 10 - 90\ ) \quad (9)$

Therefore, the r m s value of the capacity component of the impressed voltage = 400 7 × 707 = 283 3 volts, and the voltage is 90 lagging behind the current

The student should study very carefully the results of the above problem He should draw the current and voltages on a vector diagram He will note that the capacity component of the impressed voltage is in opposition to the

inductance component. Therefore their resultant will be the arithmetic difference of the two, and will $= 400.7 - 118.7 = 282$ volts maximum. This is 90° out of phase with the resistance component and therefore the final resultant, *i.e.*, the impressed voltage will be obtained by adding the two geometrically. The impressed voltage $= \sqrt{30.2^2 + 282^2}$ = approximately 283 volts. This should have been 282 volts, but the slight difference is due to the fact that in working out the above example we have utilized the Slide Rule considerably. The object of the above example was to illustrate the application of the symbolic method. The student should work out himself the above example and find out very accurately the values of current and voltage components. He should then draw the results on a vector diagram and study the diagram very critically so as to be able to form a clear mental picture of the behaviour of current and voltages.

EXAMPLE 13.—A circuit contains resistance = 4 ohms, inductance = 0.5 henries, and capacity $= 60 \times 10^{-6}$ farads, joined in series. An alternating voltage of 50 frequency is applied across the

circuit The current flowing through the circuit is expressible by

$$i = 7.56 \sin (pt + 83^\circ 10') \qquad (1)$$

Find the r m s value and phase angle of the impressed voltage

Ans — The impedance of the whole circuit

$$= R + j\left(pL - \frac{1}{pC}\right)$$

$$= 4 + j\,(15.7 - 53)$$

$$= 4 + j\,(-37.3) \qquad (2)$$

Therefore, the impressed voltage

$$e = \text{impedance} \times \text{current}$$

$$= [4 + j\,(-37.3)] \times i$$

$$= [4 + j\,(-37.3)] \times 7.56 \sin (pt + 83^\circ 10')$$

$$= \sqrt{4^2 + (-37.3)^2} \times 7.56 \sin \left(pt + 83^\circ 10' + \tan^{-1} \frac{-37.3}{4}\right)$$

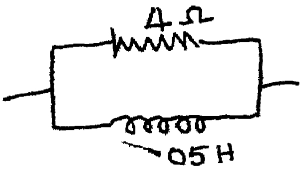
$$= \sqrt{4^2 + 37.3^2} \times 7.56 \sin (pt + 83^\circ 10' - \tan^{-1} 9.3)$$

$$= 37.3 \times 7.56 \sin (pt + 83^\circ 10' - 83^\circ 10')$$

$$= 282 \sin pt \qquad (3)$$

Therefore, the r m s value of the impressed voltage $= 282 \times .707 = 200$ volts, and it is lagging $83^\circ 10'$ behind the current

EXAMPLE 14.—A circuit comprises two branch circuits a and b joined in parallel. The branch circuit a possesses resistance = 4 ohms, and the branch circuit b contains inductance = ·05 henries. An alternating voltage of 50 ~ frequency and expressible by e = 282 sin pt is applied across the circuit. Find the r. m. s. value and the phase angle of the main current and branch currents.



ANS.—The impressed voltage =

$$e = 282 \sin pt \tag{1}$$

The impedance of branch circuit a

$$= R + j \text{ zero}$$

$$= 4 + j \text{ zero} \tag{2}$$

The impedance of branch circuit b

$$= \text{zero} + j\, p\, L$$

$$= \text{zero} + j\, 314 \times \cdot 05$$

$$= \text{zero} + j\, 15 \cdot 7 \tag{3}$$

The impedance of the whole circuit

$$= 1 \div \left(\frac{1}{R + j \text{ zero}} + \frac{1}{\text{zero} + j\, p\, L} \right)$$

$$= 1 \div \left[\frac{\text{zero} + j\, p\, L + R + j \text{ zero}}{(R + j \text{ zero})\ (\text{zero} + j\, p\, L)} \right]$$

$$= 1 - \left[\frac{R + j\,(\text{zero} + p\,L)}{(R + j\,\text{zero})\,(\text{zero} + j\,p\,L)} \right]$$

$$= \frac{(R + j\,\text{zero})\,(\text{zero} + j\,p\,L)}{R + j\,p\,L}$$

$$= \frac{(4 + j\,\text{zero})\,(\text{zero} + j\,15.7)}{4 + j\,15.7} \qquad (4)$$

Having found the symbolic expressions for various impedances, the determination of the various currents is a simple matter and will be as follows —

(*a*) The main current =

$$i = \frac{\text{impressed voltage}}{\text{impedance of the circuit}}$$

$$= \frac{282 \sin pt}{\dfrac{(4 + j\,\text{zero})\,(\text{zero} + j\,15.7)}{4 + j\,15.7}}$$

$$= \frac{(4 + j\,15.7)}{(4 + j\,\text{zero})\ (\text{zero} + j\,15.7)} \times 282 \sin pt$$

$$= \frac{\sqrt{4^2 + 15.7^2}}{\sqrt{4^2 + \text{zero}^2} \times \sqrt{\text{zero}^2 + 15.7^2}} \times 282 \sin \left(pt + \tan^{-1}\frac{15.7}{4} - \tan^{-1}\frac{\text{zero}}{4} - \tan^{-1}\frac{15.7}{\text{zero}} \right)$$

$$= \frac{16.2}{4 \times 15.7} \times 282 \sin (pt + \tan^{-1} 3.92$$

$$- \text{zero} - 90^\circ)$$

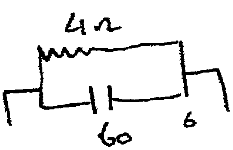
$$= 72.7 \sin (pt + 75^\circ 42' - 90^\circ)$$

$$= 72.7 \sin (pt - 14^\circ 18') \qquad (5)$$

Therefore, the r m s value of the main current is 72.7 × .707 = 51.4 amperes, and it is approximately 14° 18′ lagging behind the impressed voltage

(*b*) The currents in the branch circuits The student should work out, as an exercise, the r m s value and phase angle of currents in the branch circuits

EXAMPLE 15 — A circuit comprises two branch circuits *a* and *b*, joined in parallel The branch circuit *a* possesses resistance = 4 ohms, and the branch circuit *b* contains capacity = 60×10^{-6} farads An alternating voltage expressible by $e = 282 \sin pt$, and having a frequency of 50 cycles per second, is applied across the circuit Find the r m s value and the phase angle of the main current, and



also the r m s value and phase angle of currents in branch circuits

Ans —The impressed voltage is given by

$$e = 282 \sin pt \tag{1}$$

The impedance of the branch circuit *a*

$$= R + j \text{ zero}$$

$$= 4 + j \text{ zero} \tag{2}$$

The impedance of the branch circuit *b*

$$= \text{zero} + j \left(\frac{1}{-pC}\right)$$

$$= \text{zero} + j \left(\frac{-1}{314 \times 60 \times 10^{-6}}\right)$$

$$= \text{zero} + j \ (-53) \tag{3}$$

The impedance of the whole circuit

$$= 1 \div \left[\frac{1}{4 + j \text{ zero}} + \frac{1}{\text{zero} + j(-53)}\right]$$

$$= 1 \div \left[\frac{\text{zero} + j(-53) + 4 + j \text{ zero}}{(4 + j \text{ zero})[\text{zero} + j(-53)]}\right]$$

$$= \frac{(4 + j \text{ zero})[\text{zero} + j(-53)]}{\text{zero} + j(-53) + 4 + j \text{ zero}}$$

$$= \frac{(4 + j \text{ zero})[\text{zero} + j(-53)]}{4 + j(-53)} \tag{4}$$

Having determined the symbolic expressions for the impedances, the r m s value and phase angle of various currents will be found as follows —

(*a*) The main current

$$= \frac{\text{impressed voltage}}{\text{impedance of the circuit}}$$

$$= 282 \sin pt - \frac{(4 + j\,\text{zero})\,[\text{zero} + j\,(-53)]}{4 + j\,(-53)}$$

$$= \frac{[4 + j\,(-53)]}{(4 + j\,\text{zero})\,[\text{zero} + j\,(-53)]} \times 282 \sin pt$$

$$= \frac{\sqrt{4^2 + (-53)^2}}{\sqrt{4^2 + \text{zero}^2} \times \sqrt{\text{zero}^2 + (-53)^2}} \times$$

$$282 \sin \left(pt + \tan^{-1} \frac{-53}{4} - \tan^{-1} \frac{\text{zero}}{4} - \tan^{-1} \frac{-53}{\text{zero}}\right)$$

$$= \frac{5}{4 \times 53} \times 282 \sin (pt - \tan^{-1} 13\ 25 - \text{zero} + 90\ )$$

$$= 70\ 5 \sin (pt - 85\ 40 + 90\ )$$

$$= 70\ 5 \sin (pt + 4\ 20\ ) \qquad (5)$$

Therefore, the r m s value of the main current = 70 5 × 707 = 49 85 amperes, and the current is 4 20 in advance of the impressed voltage

(*b*) It is left to the student to work out the r m s value and phase angle of currents in branch circuits

EXAMPLE 16 — A circuit contains branch circuits *a*, *b*, and *c* joined in parallel The branch circuit *a* possesses resistance = 4 ohms, the branch circuit *b* contains inductance = 05 henries, and the branch circuit *c* contains capacity $= 60 \times 10^{-6}$ farads

An alternating voltage expressible by e = 282 sin *pt* and having a frequency of 50 cycles per second is applied across the circuit Find the r m s value and the phase angle of the main current

Ans — The voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance of the whole circuit

$$= 1 - \left\{ \frac{1}{4 + j\,\text{zero}} + \frac{1}{\text{zero} + j\,15\,7} + \frac{1}{\text{zero} + j\,(-53)} \right\}$$

$$= 1 - \left\{ \frac{j\,15\,7 \times (-53) + j\,4 \times (-53) + j\,(4 \times 15\,7)}{(4 + j\,\text{zero})\,(\text{zero} + j\,15\,7)\,[\text{zero} + j\,(-53)]} \right\}$$

$$= 1 - \left\{ \frac{(-j^2\,15\,7 \times 53) + (j\,212) + (j\,62\,8)}{(4 + j\,\text{zero})\,(\text{zero} + j\,15\,7)\,[\text{zero} + j\,(-53)]} \right\}$$

$$= 1 - \left\{ \frac{15\,7 \times 53 + j\,(-212 + 62\,8)}{(4 + j\,\text{zero})\,(\text{zero} + j\,15\,7)\,[\text{zero} + j\,(-53)]} \right\}$$

$$= \frac{(4 + j\,\text{zero})(\text{zero} + j\,15\ 7)[\text{zero} + j(-53)]}{833\ 1 + j(-149\ 2)}$$

(2)

Therefore, the main current

$$= i = \frac{\text{impressed voltage}}{\text{impedance of the circuit}}$$

$$= \frac{833\ 1 + j(-149\ 2)}{(4 + j\,\text{zero})(\text{zero} + j\,15\ 7)[\text{zero} + j(-53)]}$$

$$\times\ 282 \sin pt$$

$$= \frac{\sqrt{833\ 1^2 + (-149\ 2)^2}}{\sqrt{4^2 + \text{zero}^2} \times \sqrt{\text{zero}^2 + 15\ 7^2} \times \sqrt{\text{zero}^2 + (-53)^2}}$$

$$\times\ 282 \sin\left(pt + \tan^{-1}\frac{-149\ 2}{833\ 1} - \tan^{-1}\frac{\text{zero}}{4}\right.$$

$$\left. - \tan^{-1}\frac{15\ 7}{\text{zero}} - \tan^{-1}\frac{-53}{\text{zero}}\right)$$

$$= \frac{845}{4 \times 15\ 7 \times 53} \times 282 \sin(pt - \tan^{-1} 179$$

$$- \text{zero} - 90 + 90)$$

$$= \frac{845}{3328} \times 282 \sin(pt - 10\ 10)$$

$$= 71\ 6 \sin(pt - 10\ 10) \qquad (3)$$

Therefore, the r m s value of the main current = 71 6 × 707 = 50 62 amperes and it is lagging 10 10 behind the impressed voltage

Alternative Solution — We can solve the above problem by simplifying the symbolic expression for the impedance of the whole circuit as follows —

The voltage is given by $e = 282 \sin pt$ (1)

The impedance of the circuit

$$= 1 - \left\{ \frac{1}{4 + j\,\text{zero}} + \frac{1}{\text{zero} + j\,15\ 7} + \frac{1}{\text{zero} + j\,(-53)} \right\}$$

$$= 1 - \left\{ \tfrac{1}{4} + \frac{1}{j\,15\ 7} + \frac{1}{j\,(-53)} \right\}$$

$$= 1 - \left\{ \tfrac{1}{4} + \frac{j}{j^2\,15\ 7} + \frac{j}{j^2\,(-53)} \right\}$$

$$= 1 - [\,\tfrac{1}{4} - j\,(\ 0636\,) + j\,(\ 01884\,)\,]$$

$$= 1 - [\,\tfrac{1}{4} + j\,(-\ 04485\,)\,]$$

$$= \frac{1}{25 + j\,(-\ 04485\,)} \qquad (2)$$

Therefore, the main current

$$= \frac{\text{impressed voltage}}{\text{impedance}} = [\ 25 + j\,(-\ 04485)\,]$$

$$\times\ 282 \sin pt$$

$$= \sqrt{25^2 + (-\ 04485\,)} \times 282 \sin\left(pt + \tan^{-1}\frac{-\ 04485}{25}\right)$$

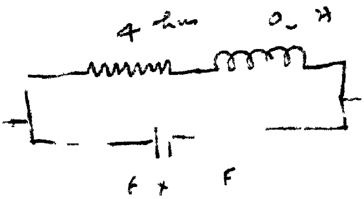
$$= 254 \times 282 \sin\left(pt - \tan^{-1} 1794\right)$$

$$= 71\ 6 \sin\left(pt - 10\ \ 10\ \right) \qquad (3)$$

Therefore, the r m s value of main current $= 71\ 6 \times 707 = 50\ 62$ amperes and it is lagging 10 10 behind the impressed voltage

The student is advised to work out the r m s value and phase angle of currents in branch circuits

EXAMPLE 17 — A circuit comprises two branch circuits a and b joined in parallel and connected across an alternating voltage supply expressible by $e = 282 \sin pt$, where $p = 2\pi \times 50$ The branch circuit a contains resistance = 4 ohms and inductance = 0.05 henries joined in series The branch circuit b possesses only capacity = 60×10^{-6} farads Find the r m s value and phase angle of main current



Answer — The impressed voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance of the whole circuit

$$= 1 \div \left\{ \frac{1}{R + jpL} + \frac{1}{\text{zero} + j\left(\frac{1}{-pC}\right)} \right\}$$

$$= 1 \div \left\{ \frac{1}{4 + j\,15.7} + \frac{1}{\text{zero} + j\,(-53)} \right\}$$

$$= 1 \div \left\{ \frac{4 + j\,(-37.3)}{(4 + j\,15.7)\,[\text{zero} + j\,(-53)]} \right\}$$

$$= \frac{(4 + j\,15.7)\,[\text{zero} + j\,(-53)]}{4 + j\,(-37.3)} \qquad (2)$$

Therefore, the main current

$$i = \frac{\text{impressed voltage}}{\text{impedance of the circuit}}$$

$$= \frac{4 + j(-37\ 3)}{(4 + j\,15\ 7)[\text{zero} + j(-53)]}\ 282 \sin pt$$

$$= \frac{\sqrt{4^2 + (-37\ 3)^2}}{\sqrt{4^2 + (15\ 7)^2} \times \sqrt{\text{zero}^2 + (-53)^2}} \times$$

$$282 \sin\left(pt + \tan^{-1}\frac{-37\ 3}{4} - \tan^{-1}\frac{15\ 7}{4} - \tan^{-1}\frac{-53}{\text{zero}}\right)$$

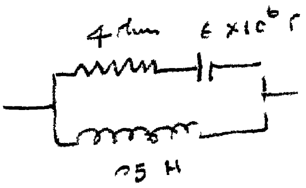
$$= \frac{37\ 5}{16\ 2 \times 53} \times 282 \sin(pt - \tan^{-1} 9\ 32 - \tan^{-1} 3\ 92 + \tan^{-1}\infty)$$

$$= 12\ 3 \sin(pt - 83°\ 50' - 75°\ 40' + 90°)$$

$$= 12\ 3 \sin(pt - 69°\ 30') \qquad (a)$$

Therefore, the r m s value of main current $= 12\ 3 \times 707 = 8\ 696$ amperes and it is 69° 30′ lagging behind the impressed voltage

Example 18 A circuit comprises two branch circuits *a* and *b* joined in parallel The branch circuit *a* possesses resistance = 4 ohms and capacity = 60×10^{-6} farads joined in series, and the branch



circuit b possesses only inductance $=$ 05 henries An alternating voltage expressible by $e = 282 \sin pt$, where $f = 50$, is applied across the circuit Find the r m s value and phase angle of main current

Answer — The impressed voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance of the whole circuit

$$= 1 - \left\{ \frac{1}{R + j\left(\frac{-1}{pC}\right)} + \frac{1}{(\text{zero} + jpL)} \right\}$$

$$= 1 - \left\{ \frac{1}{4 + j(-53)} + \frac{1}{\text{zero} + j\,15\,7} \right\}$$

$$= 1 - \left\{ \frac{4 + j(-37\,3)}{[4 + j(-53)] \times (\text{zero} + j\,15\,7} \right\}$$

$$= \frac{[4 + j(-53)]\,(\text{zero} + j\,15\,7)}{4 + j(-37\,3)} \qquad (2)$$

Therefore, the main current

$$i = \frac{\text{impressed voltage}}{\text{impedance}}$$

$$= \frac{4 + j(-37\,3)}{[4 + j(-53)]\,(\text{zero} + j\,15\,7)} \times 282 \sin pt$$

$$= \frac{37\,5}{15\,7 \times 53} \times 282 \sin\left(pt + \tan^{-1}\frac{-37\,3}{4} - \right.$$

$$\left. \tan^{-1}\frac{-53}{4} - \tan^{-1}\frac{15\,7}{\text{zero}}\right]$$

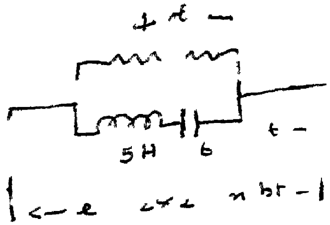
$= 12\ 7 \sin [pt - \tan^{-1} 9\ 32 + \tan^{-1} 13\ 25 -$
$\tan^{-1} \infty]$

$= 12\ 7 \sin [pt - 83\ \ 50 + 85\ \ 40 - 90\ ]$

$= 12\ 7 \sin [pt - 88\ \ 10\ ]$ (3)

Therefore, the r m s value of main current
$= 12\ 7 \times 707 = 8\ 97$ amperes, and it is lagging 88 10 behind the impressed voltage

EXAMPLE 19 A circuit comprises two branch circuits *a* and *b* joined in parallel The branch circuit *a* contains only resistance = 4 ohms, and the branch circuit *b* possesses inductance = 05 henries and capacity = 60 × 10^{-6} farads, joined in series The circuit is connected across an alternating voltage expressible by e = 282 sin *pt*, where frequency = 50 Find the r m s value and phase angle of the main current



Answer — The impressed voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance of the whole circuit

$$= 1 \div \left\{ \frac{1}{R + j\ \text{zero}} + \frac{1}{\text{zero} + j\left(pL - \frac{1}{pC}\right)} \right\}$$

$$= 1 - \left\{\frac{1}{4 + j\ \text{zero}} + \frac{1}{\text{zero} + j\,(15\ 7 - 53)}\right\}$$

$$= 1 - \left\{\frac{1}{4 + j\ \text{zero}} + \frac{1}{\text{zero} + j\,(-37\ 3)}\right\}$$

$$= 1 - \left\{\frac{4 + j\,(-37\ 3)}{(4 + j\ \text{zero})\,[\text{zero} + j\,(-37\ 3)]}\right\} \quad (2)$$

Therefore, the main current

$$i = \frac{\text{impressed voltage}}{\text{impedance}}$$

$$= \frac{4 + j - 37\ 3}{(4 + j\ \text{zero})\,[\text{zero} + j\,(-37\ 3)]} \times 282 \sin pt$$

$$= \frac{37\ 5}{4 \times 37\ 3} \times 282 \sin\left(pt + \tan^{-1}\frac{-37\ 3}{4} - \tan^{-1}\frac{\text{zero}}{4} - \tan^{-1}\frac{-37\ 3}{\text{zero}}\right)$$

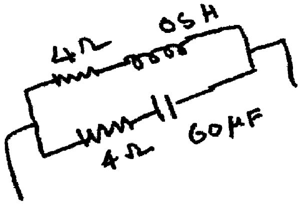
$$= 70\ 9 \sin(pt - \tan^{-1} 9\ 32 - \text{zero} + \tan^{-1}\infty)$$

$$= 70\ 9 \sin(pt - 83\ 50 + 90)$$

$$= 70\ 9 \sin(pt + 6\ 10) \quad (3)$$

Therefore, the r m s value of main current $= 70\ 9 \times 707 = 50\ 1$ amperes and it is 6 10 in advance of the impressed voltage

EXAMPLE 20 A circuit comprises two branch circuits *a* and *b* joined in parallel The branch circuit *a* contains resistance = 4 ohms and inductance = 05 henries



joined in series, and the branch circuit b possesses resistance = 4 ohms and capacity = 60×10^{-6} farads joined in series The circuit is connected across an alternating voltage expressible by e = 282 sin pt, where frequency = 50 Find the r m s value of main current and its phase angle

Answer — The impressed voltage is given by

$$e = 282 \sin pt \qquad (1)$$

The impedance of the whole circuit

$$= 1 - \left\{ \frac{1}{R + j\,pL} + \frac{1}{R + j\left(-\frac{1}{pC}\right)} \right\}$$

$$= 1 - \left\{ \frac{1}{4 + j\,15\ 7} + \frac{1}{4 + j\,(-53)} \right\}$$

$$= 1 - \left\{ \frac{8 + j(-37\ 3)}{(4 + j\,15\ 7)\,[4 + j\,(-53)]} \right\}$$

$$= \frac{(4 + j\,15\ 7)\,[4 + j\,(-53)]}{8 + j\,(-37\ 5)} \qquad (2)$$

Therefore, the main current

$$i = \frac{\text{impressed voltage}}{\text{impedance}}$$

$$= \frac{8 + j\,(-37\ 3)}{(4 + j\,15\ 7)\,[4 + j\,(-53)]} \times 282 \sin pt$$

$$= \frac{38\ 1}{16\ 2 \times 53} \times 282 \sin\left(pt - \tan^{-1}\frac{37\ 3}{8} - \tan^{-1}\frac{15\ 7}{4} - \tan^{-1}\frac{-53}{4}\right)$$

$$= 12\ 5 \sin\left(pt - \tan^{-1} 4\ 66 - \tan^{-1} 3\ 92 + \tan^{-1} 13\ 25\right)$$

$$= 12\ 5 \sin\left(pt - 77\ \ 50 - 75\ \ 40 + 85\ \ 40\right)$$

$$= 12\ 5 \sin\left(pt - 67\ \ 50\right) \qquad (3)$$

Therefore, the r m s value of main current $= 12\ 5 \times\ 707 = 8\ 83$ amperes and it is 67 50 lagging behind the impressed voltage

EXAMPLE 21 A circuit, containing resistance, inductance and capacity, arranged as shown in the diagram of Fig 18, comprises two circuits P M and M N joined in series

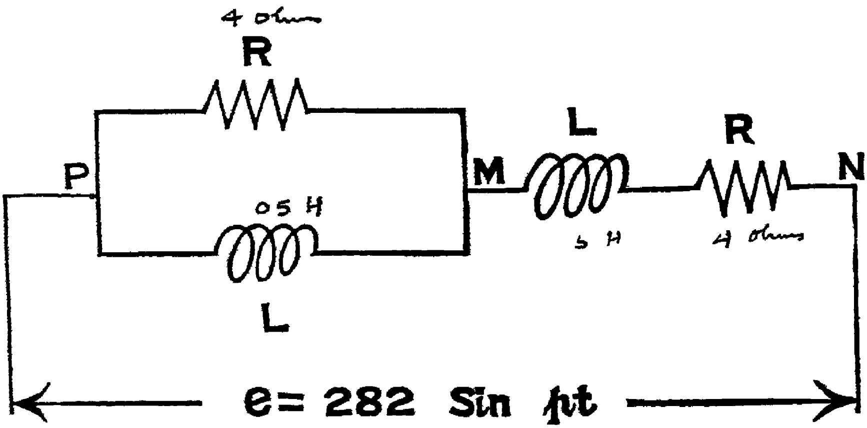


Fig: 18

The circuit P M is made up of two branch circuits joined in parallel, one containing re sistance = 4 ohms and the other containing inductance = 05 hen ries The circuit M N has in ductance = 05 henries and re sistance = 4 ohms joined in series An alternating voltage e = 282 sin *pt*, and having a frequency of 50 cycles per second, is applied across the circuit P N Find

(*a*) the r m s value and phase angle of main current

(*b*) the r m s value of currents in branch circuits of P M

Answer — (*a*) The impressed voltage

$$e = 282 \sin pt \qquad (1)$$

The impedance of the whole circuit

$$= \left\{ 1 - \frac{1}{(R + j\,\text{zero})} + \frac{1}{(\text{zero} + j\,pL)} \right\} + (R + j\,pL)$$

$$= \left\{ 1 - \frac{R + j\,pL}{(R + j\,\text{zero})\,(\text{zero} + j\,pL)} \right\} + (R + j\,pL)$$

$$= \frac{\text{zero} + j\,R\,p\,L}{R + j\,p\,L} + R + j\,p\,L$$

$$= \frac{j\,R\,p\,L\,(R - j\,p\,L)}{(R + j\,p\,L)\,(R - j\,p\,L)} + R + j\,p\,L$$

$$= \frac{j\,R\,p\,L + R\,p^2\,L}{R \;\; + p \;\; L^2} + R + j\,p\,L$$

$$= \frac{R\,p^2\,L^2}{R \;\; + p \;\; L} + \frac{j\,R\,p\,L}{R \;\; + p \;\; L} + R + j\,p\,L$$

$$= \left(R + \frac{R\,p \;\; L}{R \;\; + p \;\; L}\right) + j\left(p\,L + \frac{R \;\; p\,L}{R \;\; + p \;\; L^2}\right)$$

$$= \left(4 + \frac{4 \times 15\;7}{4^2 + 15\;7}\right) + j\left(15\;7 + \frac{16 \times 15\;7}{16 + 15\;7}\right)$$

$$= (4 + 3\;756) + j\,(15\;7 + \;\;956)$$

$$= 7\;756 + j\,16\;656 \qquad (2)$$

Therefore, the main current

$$= \frac{\text{impressed voltage}}{\text{impedance of the whole circuit}}$$

$$= \frac{282 \sin pt}{7\;756 + j\;16\;656}$$

$$= \frac{282}{\sqrt{7\;756 \;\; + 16\;656}} \sin\left(pt - \tan^{-1}\frac{16\;565}{7\;756}\right)$$

$$= \frac{282}{18\;38} \sin\left(pt - \tan^{-1} 2\;14\right)$$

$$= 15\;35 \sin\left(pt - 65\;\right) \qquad (3)$$

Therefore, the r m s value of main current

$= 15\ 35 \times\ 707 = 10\ 85$ amperes and it is 65 lagging behind the impressed voltage

(*b*) In order to find the currents in branch circuits of P M, it is necessary to know the component, of the impressed voltage, across P M The voltage across P M

= impedance of P M × main current

But, the impedance of P M

$$= 1 - \left(\frac{1}{R + j\ \text{zero}} + \frac{1}{\text{zero} + j\ p\ L} \right)$$

$$= \frac{j\ R\ p\ L}{R + j\ p\ L}$$

$$= \frac{j\ R\ p\ L\ (R - j\ p\ L)}{(R + j\ p\ L)\ (R - j\ p\ L)}$$

$$= \frac{j\ R^2\ p\ L - j^2\ R\ p\ L^2}{R\ - j\ p\ L}$$

$$= \frac{R\ p\ L + j\ R\ p\ L}{R + p\ L^2}$$

$$= \frac{4 \times 15\ 7}{4^2 + 15\ 7} + j\ \frac{4 \times 15\ 7}{4 + 15\ 7}$$

$$= 3\ 756 + j\ 956 \qquad (4)$$

Therefore, the voltage across P M

= impedance of P M × main current

$= (3\ 756 + j\ \ 956) \times 15\ 35 \sin (pt - 65\ )$

$$= \sqrt{5\ 756^2 + 956} \times 15\ 35 \ \sin\left(pt - 65 + \tan^{-1}\frac{956}{3\ 756}\right)$$

$$= 3\ 88 \times 15\ 35 \ \sin(pt - 65 + 14\ \ 11)$$

$$= 59\ 5 \sin(pt - 50\ \ 49) \qquad (5)$$

We can now determine the currents in branch circuits of P M as follows —

The current in the resistance branch of the circuit P M

$$= \frac{\text{voltage across P M}}{\text{impedance of the resistance branch}}$$

$$= \frac{59\ 5 \sin(pt - 50\ \ 49)}{R + j\ \text{zero}}$$

$$= \frac{59\ 5 \sin(pt - 50\ \ 49)}{4 + j\ \text{zero}}$$

$$= \frac{59\ 5}{\sqrt{4^2 + \text{zero}}} \sin\left(pt - 50\ \ 49 - \tan^{-1}\frac{\text{zero}}{4}\right)$$

$$= 14\ 9 \sin(pt - 50\ \ 49 - \text{zero})$$

$$= 14\ 9 \sin(pt - 50\ \ 49) \qquad (6)$$

Similarly, the current in the inductance branch of the circuit P M

$$= \frac{\text{voltage across P M}}{\text{impedance of the inductance branch}}$$

$$= \frac{59\ 5 \sin(pt - 50\ \ 49)}{\text{zero} + j\,p\,L}$$

$$= \frac{59\ 5 \sin(pt - 50\ \ 49)}{\text{zero} + j\,15\ 7}$$

$$= \frac{59\ 5}{\sqrt{\text{zero} + 15\ 7^2}} \sin\left(pt - 50\ 49 - \tan^{-1}\frac{15\ 7}{\text{zero}}\right)$$

$$= 3\ 8 \sin\left(pt - 50\ 49 - 90\right) \tag{7}$$

The r m s values of branch currents in circuit P M are 14 9 × 707 and 3 8 × 707, *i e* 10 55 and 2 68 amperes respectively

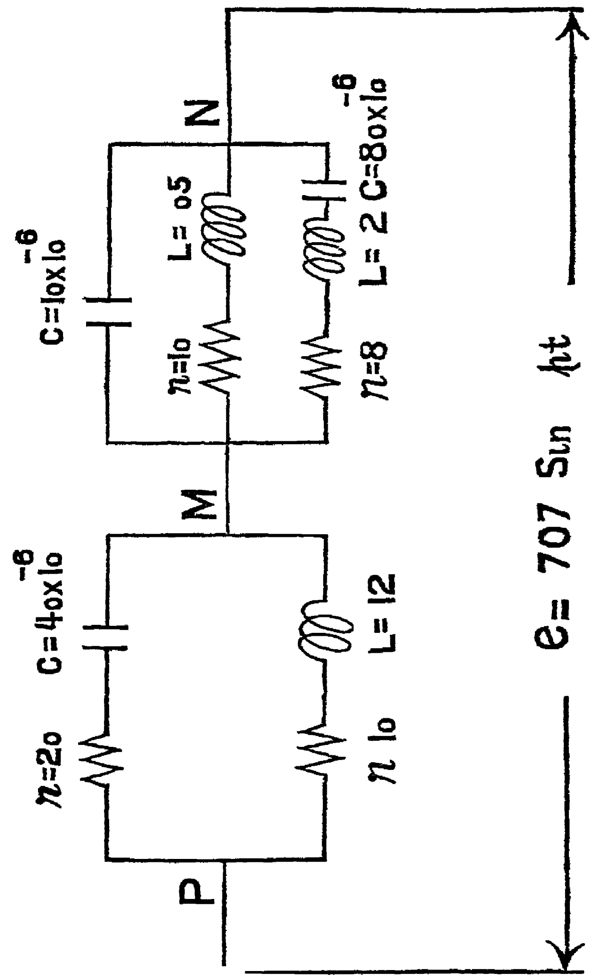


Fig 19

EXAMPLE 22 In Fig 19 is shown an

arrangement of a number of resistances, inductances and condensers. The magnitudes of these are given in the diagram. An alternating voltage of 500 volts having a frequency of 50 is applied across the extreme terminals P and N. The problem is to determine the voltage components, main current, branch currents, and their respective phase positions.

(Note.—The values in the above example have been borrowed from ALFRED HAY'S "ALTERNATING CURRENTS" page 21 22, where it has been solved analytically. We shall, however, solve it by means of the symbolic method.)

Answer.—Before we proceed to solve the problem, the attention of the student may be drawn to the value of the voltage given above. Since nothing is specified, we take it that the voltage referred to above is the r m s value. Therefore, if we want to retain the scientific character of our work we shall express the given voltage by an equation such as

$$e = E \sin pt$$

$$= \frac{500}{707} \sin pt$$

$$= 707 \sin pt \qquad (1),$$

where 707 is the maximum value and 500

is the r m s value of the applied voltage sine wave

We should first find the impedances of the various circuits as follows —

(*i*) impedance of branch *a* of circuit P M

$$= R + j \frac{1}{-pC}$$

$$= 20 + j \frac{1}{-314 \times 40 \times 10^{-6}}$$

$$= 20 - j\,79\ 6 \qquad (2)$$

(*ii*) impedance of branch *b* of circuit P M

$$= R + j\,p\,L$$

$$= 10 + j\,314 \times 12$$

$$= 10 + j\,37\ 68 \qquad (3)$$

(*iii*) joint impedance of the whole circuit P M

$$= \frac{(20 - j\,79\ 6)(10 + j\,37\ 68)}{20 + 10 + j\,(37\ 68 - 79\ 6)}$$

$$= \frac{(20 - j\,79\ 6)(10 + j\,37\ 68)}{30 - j\,41\ 92}$$

$$= 36\ 7 + j\,19\ 99 \qquad (4)$$

(*iv*) impedance of the branch *a* of circuit M N

$$= \text{zero} + j \frac{1}{-pC}$$

$$= \text{zero} + j \frac{1}{-314 \times 10 \times 10^{-6}}$$

$$= \text{zero} - j\,318 \qquad (5)$$

(*vi*) impedance of branch *b* of circuit M N

$$= R + j\,p\,L$$
$$= 10 + j\,314 \times \;05$$
$$= 10 + j\,15\;7 \tag{6}$$

(*vi*) impedance of branch *c* of circuit M N

$$= R + j\left(pL - \frac{1}{pC}\right)$$
$$= 8 + j\left(314 \times \;2 - \frac{1}{314 \times 80 \times 10^{-6}}\right)$$
$$= 8 + j\,23 \tag{7}$$

(*vii*) joint impedance of circuit M N

$$= \frac{(\text{zero} - j\,318)\,(10 + j\,15\;7)\,(8 + j\,23)}{(\text{zero} - j\,318)(10 + j\,15\;7) + (\text{zero} - j\,318)(8 + j\,23) + (10 + j\,15\;7)(8 + j\,23)}$$
$$= \frac{113\;8 + j\,89\;38}{12\;025 - j\,5\;368}$$
$$= \frac{(113\;8 + j\,89\;38)\,(12\;025 + j\,5\;368)}{(12\;025 - j\,5\;368)\,(12\;025 + j\,5\;368)}$$
$$= 5\;1 + j\,9\;69 \tag{8}$$

The joint impedance of circuit M N can also be worked out in a much simpler way as follows —

admittance of branch *a* of circuit M N

$$= \frac{1}{\text{zero} - j\,318} = \text{zero} + j\;00314,$$

admittance of branch *b* of circuit M N

$$= \frac{1}{10 + j\,15\;7} = \;0289 - j\;0454,$$

admittance of branch c of circuit M N

$$= \frac{1}{8 + j\,23} = 0155 - j\,0587$$

joint admittance of circuit M N $= 0424 - j\,0809$

joint impedance of M N $= \dfrac{1}{0424 - j\,0809}$

$$= \frac{0424 + j\,0809}{(0424)^2 + (0809)^2}$$

$$= 5\,1 + j\,9\,69 \qquad (8)$$

(*viii*) the impedance of the whole circuit P N

= joint impedance of P M + joint impedance of M N

$= 36\,7 + j\,49\,99 + 5\,1 + j\,9\,69$

$= 41\,8 + j\,59\,68$ (9)

Having found the symbolic expressions for the impedances of all the circuits, the determination of currents and voltages, entering into the problem, becomes quite a simple matter It will be worked out as follows —

(*A*) Main current

$$= \frac{\text{voltage across P N}}{\text{impedance of P N}}$$

$$= \frac{707 \sin pt}{41\,8 + j\,59\,68}$$

$$= \frac{707}{\sqrt{41\,8^2 + 59\,68^2}} \sin\left(pt - \tan^{-1}\frac{59\,68}{41\,8}\right)$$

$$= \frac{707}{72\ 8} \sin\ (pt - \tan^{-1} 1\ 42\ )$$

$$= 9\ 72 \sin\ (\ pt - 55\ ) \qquad (10)$$

Therefore, the r m s value of the main current

$$= 9\ 72 \times\ 707 = 6\ 87 \text{ amperes,}$$

and it lags 55 behind the applied voltage

(*B*) Voltage component across P M

$$= \text{main current} \times \text{joint impedance of P M}$$

$$= 9\ 72 \sin\ (\ pt - 55\ ) \times (\ 36\ 7 + j\ 49\ 99\ )$$

$$= 9\ 72 \times \sqrt{36\ 7^2 + 49\ 99^2} \sin\ (\ pt - 55\ + \tan^{-1}\frac{49\ 99}{36\ 7}\ )$$

$$= 602\ 6 \sin\ (\ pt - 55\ + 53\ \ 45\ ) \qquad (11)$$

Therefore, the r m s value of the voltage com ponent across P M

$$= 602\ 6 \times\ 707 = 426 \text{ volts}$$

and it is 53 45 in advance of the main current and 1 15 lagging behind the impressed voltage

(*C*) Voltage component across M N

$$= \text{main current} \times \text{joint impedance of M N}$$

$$= 9\ 72 \sin\ (\ pt - 55\ ) \times (\ 5\ 1 + j\ 9\ 69\ )$$

$$= 9\ 72 \times \sqrt{5\ 1^2 + 9\ 69^2} \sin\ (\ pt - 55\ + \tan^{-1}\frac{9\ 69}{5\ 1})$$

$$= 106 \sin\ (\ pt - 55\ + 62\ \ 15\ ) \qquad (12)$$

Therefore, the r m s value of the voltage com ponent across M N

$= 106 \times 707 = 75$ volts,

and it is 62 15 in advance of the main current and 6 45 in advance of the impressed voltage It is about 8 in advance of the voltage component across P M

(*D*) Current in branch *a* of circuit P M

= voltage across P M – impedance of the branch circuit

$$= \frac{602\ 6 \sin (pt - 1\ 15\ )}{20 - j\ 79\ 6}$$

$$= \frac{602\ 6}{\sqrt{20^2 + 79\ 6^2}} \sin (pt - 1\ 15\ - \tan^{-1}\frac{-79\ 6}{20})$$

$$= 7\ 28 \sin (pt - 1\ 15\ + 75\ 50\ ) \qquad (15)$$

Therefore, the r m s value of current in branch *a* of circuit P M

$= 7\ 28 \times 707 = 5\ 15$ amperes,

and it is 75 50 in advance of the voltage across P M

(*E*) Current in branch *b* of circuit P M

= voltage across P M – impedance of branch *b*

$$= \frac{602\ 6 \sin (pt - 1\ 15\ )}{10 + j\ 37\ 68}$$

$$= \frac{602\ 6}{\sqrt{10^2 + 37\ 68^2}} \sin (pt - 1\ 15\ - \tan^{-1}\frac{37\ 8}{10})$$

$$= 15\ 41 \sin (pt - 1\ 15\ - 75\ 5\ ) \qquad (14)$$

Therefore, the r m s value of current in branch b of circuit P M $= 15\ 41 \times 707 = 10\ 9$ amperes, and it is 75 5 lagging behind the voltage across P M

(F) Current in branch a of circuit M N

$$= \text{voltage across M N} - \text{impedance of the branch circuit}$$

$$= 106 \sin [\,pt + 6\ 45\ ] - [\,\text{zero} - j\ 318\,]$$

$$= \frac{106}{\sqrt{\text{zero} + 318}} \sin \left[\,pt + 6\ 45 - \tan^{-1} \frac{-318}{\text{zero}}\right]$$

$$= 333 \sin [\,pt + 6\ 45 + \tan^{-1} \infty\ ]$$

$$= 333 \sin [\,pt + 6\ 45 + 90\ ] \qquad [15]$$

Therefore, the r m s value of current in branch circuit a of M N $= 333 \times 707 = 235$ amperes and it is 90 in advance of the voltage across M N

$[G]$ Current in branch b of circuit M N

$$= \text{voltage across M N} - \text{impedance of the branch circuit}$$

$$= 106 \sin [\,pt + 6\ 45\ ] - [\,10 + j\ 15\ 7\,]$$

$$= \frac{106}{\sqrt{10^2 + 15\ 7}} \sin \left[\,pt + 6\ 45 - \tan^{-1} \frac{15\ 7}{10}\right]$$

$$= 5\ 7 \sin [\,pt + 6\ 45 - 57\ 30\ ] \qquad [16]$$

Therefore, the r m s value of current in branch b of circuit M N $= 5\ 7 \times 707 = 4\ 03$ amperes and it is 57 30 lagging behind the voltage across M N

[*H*] Current in branch *c* of circuit M N

= voltage across M N – impedance of the branch circuit

$$= 106 \sin [\, pt + 6\ 45 \,] - [\, 8 + j\, 23 \,]$$

$$= \frac{106}{\sqrt{8^2 + 23^2}} \sin \left[\, pt + 6\ 45 - \tan^{-1} \frac{23}{8} \,\right]$$

$$= 4\ 36 \sin [\, pt + 6\ 45 - 70\ 45 \,] \qquad [17]$$

Therefore, the r m s value of current in branch *c* of circuit M N = 4 36 × 707 = 3 08 amperes and it is 70 45 lagging behind the voltage across M N

We can now summarise, in a tabular form, the above results as follows —

TABLE I

Item No	Particulars	Analytical expression	Maximum value
1	Impressed voltage	e = 707 sin *pt*	707
A	Main current	i = 9 72 sin (*pt* −55)	9 7
B	Voltage across P M	e = 602 6 sin (*pt* −1 15)	602 6
C	Voltage across M N	e = 106 sin (*pt* +6 45)	106
D	Current in branch *a* of circuit P M	i = 7 28 sin (*pt* +74 35)	7 28
E	Current in branch *b* of circuit P M	i = 15 41 sin (*pt* −76 20)	15 41
F	Current in branch *a* of circuit M N	i = 333 sin (*pt* +6 45 +90)	333
G	Current in branch *b* of circuit M N	i = 5 7 sin (*pt* −50 45)	5 7
H	Current in branch *c* of circuit M N	i = 4 36 sin (*pt* −64)	4 36

R M S value	Phase angle with respect to standard vector of reference	Phase angle with respect to voltage across P M	Phase angle with respect to voltage across M N	Remarks
500	Zero	—	—	The impressed voltage is our standard vector of reference lying in positive horizontal direction
6 87	Lags 55	—	—	
426	Lags 1 15	—		
75	Leads 6 45	—		
5 15		Leads 75 50		
10 9		Lags 75 5		
235		—	Leads 90	
4 03		—	Lags 57 30	
3 08		—	Lags 70 45	

A glance at the values given in the foregoing table will show that

[i] the arithmetic sum of voltages across P M and M N is nearly equal to the impressed voltage Such a result is purely accidental and is due to very small difference in the respective phase positions

[ii] Although the main current is 6 87 amperes, the current in one of the branch circuits is 10 9 amperes Further, the arithmetic sum of branch currents of either of P M or of M N is greater than the main current

The student should think out for himself as to why the current in a branch circuit is considerably greater than the main current, and the arithmetic sum of the branch currents of either P M or M N is greater than the main current

INDEX

(Th r f to p e)